Authors' Response to Strasberg's "Comment on 'Measurement of the frequency dependence of the ultrasonic parametric threshold amplitude for a fluid-filled cavity' " [J. Acoust. Soc. Am. 125, 1857 (2009)] (L)

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This letter is a response to Strasberg's recent paper, "Comment on 'Measurement of the frequency dependence of the ultrasonic parametric threshold amplitude for a fluid-filled cavity.'" The authors dispute the conclusions of Strasberg regarding the effect observed by Teklu *et al.* [J. Acoust. Soc. Am. **120**, 657–660 (2006)] published previously in JASA.
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We would like to thank Strasberg for bringing attention to the frequency dependence of acoustical attenuation. At first we thought parametrically excited fractional harmonics would be a way to measure this attenuation.[1] We were aware of the dependence pointed out by Strasberg at the time of our 2006 paper, but rejected the situation described by Strasberg in favor of a more complete explanation based on the solutions of Mathieu's equation, which is appropriate for our system. The behavior of the solutions is depicted in the stability chart for Mathieu functions published by McLachlan.[2] This stability chart is unusual in that the coordinates are not independent, but if properly used, it accounts for the dependence of the threshold amplitude on cavity length. The curve beginning at $a=1$ corresponds to data taken by Adler in 1970; that beginning at $a=4$ corresponds to the 2006 data.[3,4] Subharmonics occur when the threshold line between unstable and stable regions labeled in the curve is crossed. In the experiments we described, these curves were crossed near $q=0$. The presence of such parametrically excited subharmonics is demonstrated optically by the appearance of extra diffraction orders between the ordinary diffraction orders when the transducer drive voltage is above a threshold value. It has been stated in the 2006 paper that parametric resonance is observed only if the energy supplied to the system reaches a critical value large enough to overcome attenuation in the system, mathematically interpreted as Eq. (12) in the 2006 paper. When the cavity length is large, the threshold amplitude is influenced by nonlinearity of the medium, acoustic attenuation, beam spreading, or detuning, or any combinations of these, and results in the decrease in the threshold amplitude of subharmonics with the increase in the drive frequency as shown in Fig. 3 of the 2006 paper. However, if the cavity length is small, energy losses are dominated by attenuation and result in the frequency dependence of the threshold amplitude shown in Fig. 4 of the 2006 paper. It is worth mentioning that a linear system is sufficient to excite parametric resonance.

Strasberg's objection to the lack of the $\frac{1}{2}$ harmonic subfrequency in the spectrum can be explained by the existence of frequency doublets, which appear around the half frequency. The fact that doublets were observed has been explained by Eller.[5] The doublets occur because two coupled parametric oscillators are excited. Presumably the two surfaces of the reflector are coupled to produce the doublets; however, this hypothesis has not been conclusively confirmed.

The thresholds calculated by Strasberg evidently were made by assuming that the signal outputs of the transducers used in the 1970 and 2006 papers are the same, or at least of the same order of magnitude. However, in the 2006 paper, a sinusoidal wave from a function generator via a power amplifier with impedance matching network was applied to *X*-cut quartz transducers. We are not quite sure how power was supplied to the transducer in the 1970 paper. Therefore, unless we exactly measure the power output signal of the transducers at subharmonic frequencies, we cannot compare the vibration amplitudes of the transducers used in the two different experiments.

We appreciate the diligence of Strasberg. However, we do not agree with his conclusion that the spectra may not be caused by acoustical parametric oscillation. Acoustical parametric oscillation also is a reasonable explanation of the data of Yen.[6]

[1]L. Adler and M. A. Breazeale, "Measurement of ultrasonic attenuation from the threshold of parametrically excited fractional harmonics in a liquid," J. Phys. D: Appl. Phys. **4**, L3–L4 (1971).
[2]N. W. McLachlan, *Theory and Application of Mathieu's Function* (Oxford University Press, Oxford, 1947), p. 40.

[3]A. Teklu, M. S. McPherson, M. A. Breazeal, R. D. Hasse, and N. F. Declercq, "Measurement of the frequency dependence of the ultrasonic parametric threshold amplitude for a fluid-filled cavity," J. Acoust. Soc. Am. **120**, 657–660 (2006).
[4]M. S. McPherson, M. A. Breazeale, and A. Teklu, "Stability zones and acoustic parametric oscillation," official publication of the 19th International Congress on Acoustics, Madrid, CD ed. (2007).
[5]A. I. Eller, "Fractional-harmonic frequency pairs in nonlinear systems," J. Acoust. Soc. Am. **53**, 758–765 (1973).
[6]N. Yen, "Experimental investigation of subharmonic generation in an acoustic interferometer," J. Acoust. Soc. Am. **57**, 1357–1362 (1975).