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On the Linearity of Leaky Lamb Waves' Damping Dependence on the Impedance Difference Between Two Liquids Separated by a Thin Solid Plate

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Summary

When a plate is separating two different water-like liquids, the real part of the ultrasonic Lamb wave pole is almost independent of the liquids, whereas the imaginary part highly depends on the physical difference between the two liquids. It is found that this dependence possesses linear characteristics if both liquids are realistic. If one of the liquids is unrealistic, then the dependence does not obey this linearity anymore. The study is limited to the vast majority of liquids having an acoustic impedance comparable to that of water and enables a deterministic resolution of the reliability of measured or assumed liquid properties.

1. Introduction

Characterization of liquids by means of ultrasound has gained importance due to tightened port and airport security. An earlier study [1] revealed susceptibility of the Schoch effect, caused by Lamb waves, to characteristics of liquids in closed containers. Deciding whether the physical properties of a liquid (after measurements or after assumptions) are realistic or not is difficult and often not deterministic.

2. Method

In what follows, it will be shown that the imaginary part of the complex pole corresponding to Lamb wave generation in solid plates, depends on the impedance difference between the two liquids on both sides of the plate. Furthermore, it will be shown that this dependency is linear for all realistic liquids, but that it is not linear and even random, for the case when one of the liquids is unrealistic. Even though the physical phenomenon behind the effect is a complicated involvement of density and sound velocity, a plot in terms of impedance (density × velocity) results in the mentioned clear difference in behavior between realistic and unrealistic liquids.

In this report, we consider 3 solids: stainless steel (density $\rho = 7900\,\mathrm{kg/m^3}$, shear wave velocity $v_s = 3200\,\mathrm{m/s}$, longitudinal wave velocity $v_l = 5790\,\mathrm{m/s}$), brass ($\rho = 8600\,\mathrm{kg/m^3}$, $v_s = 2150\,\mathrm{m/s}$, $v_l = 4410\,\mathrm{m/s}$) and glass ($\rho = 2500\,\mathrm{kg/m^3}$, $v_s = 3520\,\mathrm{m/s}$, $v_l = 5660\,\mathrm{m/s}$).

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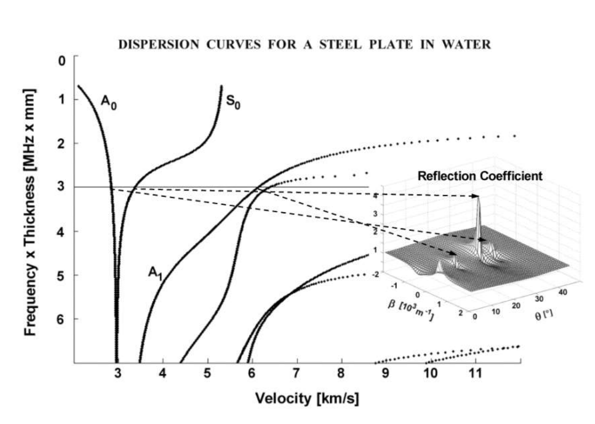

Figure 1. The dispersion curves for Lamb waves in a stainless steel plate are given. The added horizontal line corresponds to a $f \times d = 3$ MHz×mm. Three types of Lamb waves are indicated: A_0, S_0 and A_1 Lamb waves. A small image is added of the reflection coefficient for this case as a function of the real angle of incidence θ and the inhomogeneity β in the case where the plate is immersed in water. For each type of Lamb wave, there is a position $[\theta, \beta]$ where the reflection coefficient corresponds to a pole or singularity.

For a plate in vacuum, Lamb waves are solutions of the equations, expressing the disappearance of normal stress at the vacuum/solid interfaces. The solutions are real. For a plate immersed in a liquid, Lamb waves are solutions of the continuity equations, expressing continuity of normal stress and normal displacements at the liquid/solid interfaces. Here, the solutions are complex, because Lamb waves radiate energy into the liquid, resulting in a complex wave vector of its plane wave components. The complex solutions correspond to poles, or singularities, of the reflection coefficient for inhomogeneous plane waves incident from the liquid onto the plate. We apply the theoretical development of [1] and extract information about the position of those poles.

The generalized Snell's law [3, 4, 5, 6, 7] implies that the complex wave vector component parallel to the plate is, for a given liquid (liquid 1) from which sound is incident, completely determined by the real angle of incidence θ and the inhomogenity vector β; the latter is defined as the reversed imaginary component of the wave vector. A (complex) pole of the reflection coefficient corresponding to leaky Lamb waves, determined by this real angle θ and inhomogenity vector β.

We noticed, in agreement with the fact that the Lamb wave velocity is not significantly influenced by the surrounding liquid(s), that, for a given 'liquid 1', the real angle is independent of the liquid on the other side of the plate (liquid 2). Therefore, the value of the real angle of incidence θ is applied to determine the type of leaky Lamb wave that is generated (A_0, S_0, A_1, S_1, …), whereas the inhomogeneity value β reveals information about liquid 2.

3. Results

In Figure 1, the dispersion curves for Lamb waves in a stainless steel plate are given. We consider a [frequency × thickness] equal to $f \times d = 3$ MHz×mm. Three types Lamb waves are indicated, i.e. A_0, S_0 and A_1. Moreover, the reflection coefficient in this case is shown as a function of the real angle of incidence θ and the

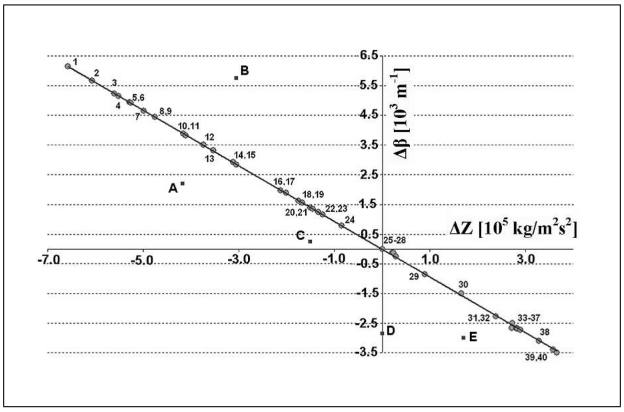

Figure 2. The inhomogeneity difference $\Delta\beta = \beta - \beta_0$ as a function of the impedance difference $\Delta Z = Z - Z_0$. The results for the stainless steel plate, in the case of A_1 Lamb wave stimulation, for 'liquid 1' = water and 'liquid 2' listed in Table I. The codes corresponding to the different liquids are tagged.

Table I. List of liquids, in order of increasing acoustical impedance. We considered 40 liquids. The list is limited to a selection of 15 liquids.

Tag	Name	ρ [kg/m^3]	C [m/s]
2	Alcohol, methanol	791	1103
A	Unrealistic liquid A	147	6750
8	Gasoline	803	1250
B	Unrealistic liquid B	309	3440
10	Petroleum	825	1290
16	Fish Oil	880	1440
20	Soybean Oil	930	1430
C	Unrealistic liquid C	465	2860
25	Water	1000	1480
D	Unrealistic liquid D	400	3700
26	Silicon dow oil	1110	1352
E	Unrealistic liquid E	568	2900
31	Glycol-PE 400	1060	1620
36	Glycol-diethylene	1116	1580
40	Glycol-ethylene	1113	1658

inhomogeneity β in the situation where the plate is immersed in water. Note that for each type of Lamb wave, there is a position $[\theta, \beta]$ where the reflection coefficient corresponds to a pole or singularity. Table I lists a selection of the different liquids that are considered. Because the number of possible unrealistic liquids is infinite, only 5 of them are listed (A–E). As a reference, each time, we take the considered plate immersed in water (liquid 1 = liquid 2) and determine the inhomogeneity β_0 at which the pole exists (for the given impedance Z_0 of water).

Then, if liquid 2 changes, we determine the corresponding values β and Z, and plot $\Delta\beta = \beta - \beta_0$ as a function of $\Delta Z = Z - Z_0$. The results for the stainless steel plate, in the case of A1 Lamb wave stimulation, are shown in Figure 2. The codes corresponding to the different liquids are tagged and are omitted in the subsequent figures; they can always be reconstructed because the numbers follow the impedance sequence of Table I. Note that the values corresponding to existing liquids all follow a linear tendency, whereas the values corresponding to unrealistic liquids are distributed within the coordinate plane. In fact, a larger number of calculations showed that the more unrealistic the properties of a liquid are, the further away the result ends up from the straight line. The straight lines depicted in our figures are obtained through a least squares approximation. In Figure 3, the same procedure was followed for different plate materials (resembling a container skin) and only for existing (realistic) liquids. The results follow straight lines once again and the direc-

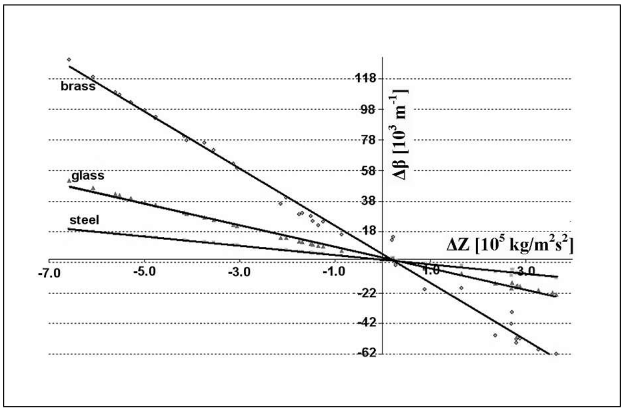

Figure 3. The same procedure was followed as in Figure 2, but for different materials of the solid plate (resembling a container skin) and only for experimentally existing liquids. Note that the results follow straight lines and the direction of each straight line is a function of the solid under consideration.

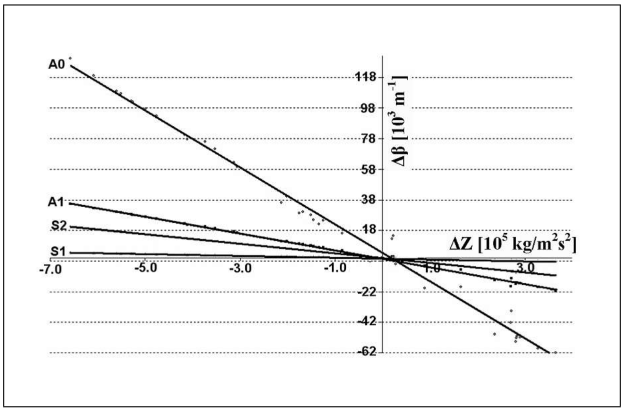

Figure 4. A brass plate is considered under the same circumstances as in Figure 2 and Figure 3, though for different Lamb waves. The direction depends on the type of Lamb wave.

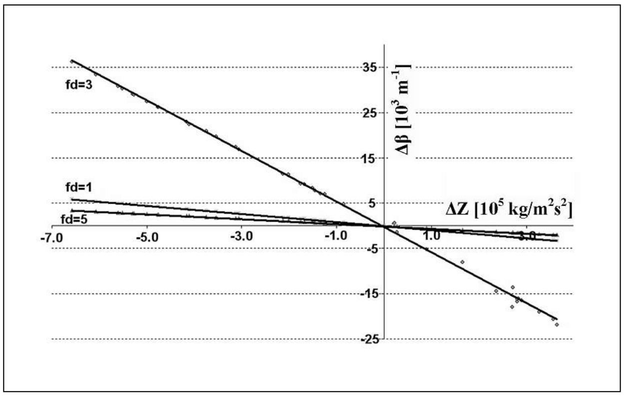

Figure 5. The result is shown for other combinations of $f \times d$. Again, the existing liquids follow a linear tendency with a direction depending on $f \times d$.

tion of each straight line is a function of the solid under consideration. In Figure 4, the case of a brass plate is considered under the same circumstances as in Figure 2 and Figure 3, though for different Lamb waves. The direction appears to be dependent on the type of Lamb wave. Finally in Figure 5, the result is shown for other combinations of $f \times d$. Again, the existing (realistic) liquids follow a linear tendency with a direction depending on $f \times d$.

4. Conclusions

It is shown that the imaginary part of the complex pole of the reflection coefficient, denoted by the inhomogeneity value â is linearly dependent of the impedance difference between 'liquid 1' and 'liquid 2' surrounding a solid plate. This linear tendency depends on the kind of Lamb wave, on the considered frequency and on the material constituting the solid plate. Furthermore, the linear trend only holds for realistic liquids. Unrealistic liquids do not follow this linear trend and produce randomly scattered results. Additional (omitted) calculations, indicate that similar conclusions hold for visco-elastic liquids. The discourse has been limited to liquids having an impedance similar to that of water (as most of the liquids do).

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