Acta Acustica united with Acustica
Vol. 92 (2006) 976 – 981

A Double Leaky Type of Surface Wave on Brass Immersed in Water

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Summary
The inhomogeneous wave theory has proved to be well suited to predict and to describe properties and the excitation of surface waves such as the leaky Rayleigh wave. The theory is applied here for a water/brass interface and shows that a new kind of leaky surface waves can exist that radiates both into the liquid and into the solid. Properties such as polarization, propagation and amplitude distribution are described and systematically compared with features of leaky Rayleigh waves.

PACS no. 43.20.El, 43.20.Bi, 43.20.Hq

1. Introduction

A surface wave can be defined as a wave whose spatial amplitude distribution contains a lobe (or amplitude concentration) along the interface. For Scholte – Stoneley waves the 'lobe', in the solid as well as in the liquid, is simply an exponentially decreasing function and it does not leak any energy. For leaky Rayleigh waves [1, 2, 3, 4], the 'lobe' is a small region in the solid, close to the surface, which emits or 'leaks' its energy into the liquid, therefore having a decreasing amplitude as a function of the traveled distance. The new kind of surface waves that is discussed here, contains a 'lobe' in the solid close to the interface, turning into a bulk wave at larger distances due to energy emission into the solid, and it also leaks energy into the liquid. As far as we know, there is only one paper that discusses a similar phenomenon at a solid/vacuum interface, i.e. a paper by Nesvijski [5]. Nesvijski [5] has discovered that the Rayleigh equation shows some poles in addition to the well known Rayleigh poles that correspond to situations under which waves may exist that show a lobe near the surface and turn into a bulk wave at larger distances. Nesvijski [5] also discusses some important applications of this phenomenon. However, since it is usual in non destructive testing to couple a transducer to a solid by means of a liquid, the impact of such a solid/vacuum special surface wave, though scientifically very interesting, in most of the applications might not be that considerable. Just as the classical Rayleigh wave has become really important in non destructive testing when its leaky counterpart [1, 2, 3, 6] was discovered and applied [7, 8, 9, 10, 11, 12, 13], the new kind of surface wave of Nesvijski [5] might only become significant due to the existence of its leaky counterpart which is discussed here. The generation of such new kind of leaky surface waves must be taken into account when new models are constructed or experiments are performed to study the generation of surface waves by impinging sound [14, 15, 16, 17] or even by laser light [18, 19].

Numerical evidence of the existence of the new type of surface wave was found, not when considering poles of the Rayleigh equation or the Scholte-Stoneley equation [20, 21], but when calculating the reflection coefficient as a function of the angle of incidence and the inhomogeneity of impinging inhomogeneous waves.

The reflection coefficient for a brass/water interface shows a peak in addition to the well known leaky Rayleigh wave peak. We have compared its location with those of the poles and branch cuts that are discussed by Pott and Harris [15, 16]. They distinguish between the four surface wave poles (two for leaky Rayleigh waves and two

Received 7 June 2005,
accepted 23 June 2006.

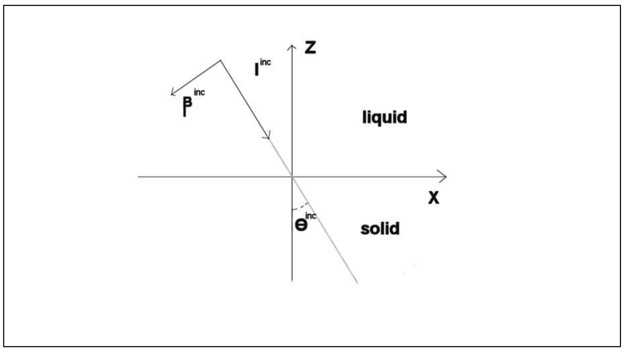

Figure 1. The real part $\boldsymbol{l}^{\text{inc}}$ of a complex incident wave vector $\boldsymbol{k}^{\text{inc}}$ together with the inhomogeneity $\boldsymbol{\beta}^{\text{inc}}$.

for Scholte – Stoneley waves) and four 'branch cuts', that give rise to lateral waves (two for shear lateral waves and two for longitudinal lateral waves). The latter can be found experimentally in the article of Couchman and Bell [22]. The new kind of surface waves as considered here, is different from phenomena resulting from these eight critical points.

In what follows, we fully apply the properties of inhomogeneous waves, which can be found in numerous articles [23, 24, 25, 26, 27, 28, 29].

In our calculations, we apply the Deschamps' principles [30, 31] for the appropriate choice of the sign of scattered wave vectors. Furthermore, we show numerical results and illustrate some calculated properties of the new kind of leaky surface waves and compare them with the ones of leaky Rayleigh waves.

2. The scattering of inhomogeneous waves

2.1. The wave potentials, the application of the generalized Snell's law and Deschamps' principles

We consider 2 dimensional scattering of sound at an interface along the x-direction as depicted in Figure 1.

The generalized Snell's law [23, 24, 25, 26, 27, 28, 29] involves continuity of the x-component of the complex valued wave vector $\boldsymbol{k}$. As a result, the potential for the incident wave is described by

$$\varphi^{\text{inc}} = \exp\left(\mathrm{i}k_x x + \mathrm{i}k_z^{\text{inc}} z\right), \tag{1}$$

and the reflected, the transmitted longitudinal and the transmitted shear respectively by

$$\varphi^{\text{r}} = RD \exp\left(\mathrm{i}k_x x + \mathrm{i}k_z^{\text{r}} z\right), \tag{2}$$
$$\varphi^{\text{t}} = TD \exp\left(\mathrm{i}k_x x + \mathrm{i}k_z^{\text{td}} z\right), \tag{3}$$
$$\psi^{\text{t}} = TS \exp\left(\mathrm{i}k_x x + \mathrm{i}k_z^{\text{ts}} z\right)\boldsymbol{e}_y, \tag{4}$$

with

$$\boldsymbol{k}^{\xi} = k_x \boldsymbol{e}_x + k_z^{\xi} \boldsymbol{e}_z = \boldsymbol{l}^{\xi} + \mathrm{i}\boldsymbol{\alpha} - \mathrm{i}\boldsymbol{\beta}, \tag{5}$$

with ξ =inc, r, td, ts, and with $\boldsymbol{l}^{\xi}$ being the real part and $\boldsymbol{\alpha}^{\xi} - \boldsymbol{\beta}^{\xi}$ the imaginary part of the complex wave vector $\boldsymbol{k}$. $\boldsymbol{\beta}^{\xi}$ is called the inhomogeneity vector [23, 24, 25, 26, 27, 28, 29], while $\boldsymbol{\alpha}^{\xi}$ is called the damping vector [23, 24, 25, 26, 27, 28, 29] and $\boldsymbol{l}^{\xi} \perp \boldsymbol{\beta}^{\xi}$ while $\boldsymbol{l}^{\xi} || \boldsymbol{\alpha}^{\xi}$.

In Figure 1, we have depicted $\boldsymbol{l}^{\text{inc}}$ and $\boldsymbol{\beta}^{\text{inc}}$. Furthermore, one may write

$$\begin{bmatrix} k_x \\ k_z^{\text{inc}} \end{bmatrix} = \begin{bmatrix} \cos\theta^{\text{inc}} & \sin\theta^{\text{inc}} \\ -\sin\theta^{\text{inc}} & \cos\theta^{\text{inc}} \end{bmatrix} \cdot \frac{-\mathrm{i}\beta^{\text{inc}}}{\sqrt{(\omega/v + \mathrm{i}\alpha_0)^2 - (\beta^{\text{inc}})^2}}, \tag{6}$$

where β^{inc} is the inhomogeneity of the incident inhomogeneous wave, where θ^{inc} is the angle of incidence, and where α_0, α_{0d} and α_{0s} are the intrinsic damping coefficients for the liquid respectively for the longitudinal waves in the solid and the shear waves in the solid.

From the dispersion relation for inhomogeneous waves [23, 24, 25, 26, 27, 28, 29], we know that

$$k_z^{\text{r}} = \pm\sqrt{(\omega/v + \mathrm{i}\alpha_0)^2 - k_x^2}, \tag{7}$$
$$k_z^{\text{td}} = \pm\sqrt{(\omega/v_d + \mathrm{i}\alpha_{0d})^2 - k_x^2}, \tag{8}$$
$$k_z^{\text{ts}} = \pm\sqrt{(\omega/v_s + \mathrm{i}\alpha_{0s})^2 - k_x^2}. \tag{9}$$

The wave velocity in the liquid is v, the longitudinal respectively shear wave velocities in the solid are v_d and v_s. The angular frequency is ω.

The signs in (7)–(9) must be chosen according to Deschamps' principles, in order to correspond to experimentally verified properties [30, 31].

Deschamps' principles state that if $\Re e\{k_z\} = 0$, Scholte-Stoneley-like surface modes are involved and the sign of k_z must be so that there is exponential decay of the amplitude away from the interface, i.e. the Sommerfeld conditions must hold. If $\Re e\{k_z\} \neq 0$, then the sign of k_z depends on the angle of propagation of the liquid-side companion of the considered longitudinal or shear mode in the solid

$$|\theta| = \arctan\left[|k_x|/|k_z^{\text{r}}|\right]. \tag{10}$$

Then, if

$$|\theta^{\text{p}}| = \arctan\left[|k_x|/|k_z^{\text{p}}|\right] \tag{11}$$

is 'close enough' to $\pi/2$, then that particular mode 'p' (p=d for longitudinal waves, p=s for shear waves) must show leaky Rayleigh wave features, whence the inhomogeneity vector $\boldsymbol{\beta}$ must point into the liquid. Deschamps [31] states that 'close enough' to $\pi/2$ means that the liquid side companion must fulfill

$$|\theta| > \left|\arcsin[v/v_p]\right|. \tag{12}$$

Otherwise, whenever

$$|\theta| \leq \left|\arcsin[v/v_p]\right|, \tag{13}$$

the Sommerfeld conditions hold, demanding that the mode 'p' travels away from the interface.

2.2. The continuity conditions and the expression for the reflection coefficient

The strain tensor is given by

$$\varepsilon_{i,j} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right), \tag{14}$$

with displacement vector

$$\boldsymbol{u} = \nabla\varphi \tag{15}$$

in the liquid and

$$\boldsymbol{u} = \nabla\varphi + \nabla \times \psi \tag{16}$$

in the solid for which the respective potentials are given in (1)–(4).

If we denote the stress tensor by $T_{i,j}$, then

$$T_{i,j} = \delta_{i,j}\tilde{\lambda}\varepsilon_{n,n} + 2\tilde{\mu}\varepsilon_{i,j}, \tag{17}$$

in which $\delta_{i,j}$ is the Kronecker delta, and $\tilde{\lambda}$ and $\tilde{\mu}$ are the complex Lamé constants.

The continuity of normal stress and displacement (upper index 1: liquid, 2: solid) along the interface are respectively given by

$$T^1_{m,3} = T^2_{m,3}, \qquad m = 1, 2, 3, \tag{18}$$

and

$$u^1_z = u^2_z. \tag{19}$$

Relations (18)–(19) result in the following continuity condition:

$$\begin{bmatrix} RD \\ TD \\ TS \end{bmatrix} = \begin{bmatrix} k_z^{\mathrm{r}} & -k_z^{\mathrm{td}} & -k_x^{\mathrm{inc}} \\ 0 & -2k_z^{\mathrm{td}}k_x & (\frac{\omega}{v} + \mathrm{i}\alpha_{0s})^2 - 2(k_x)^2 \\ -\tilde{\lambda}^1(\frac{\omega}{v} + \mathrm{i}\alpha_0)^2 & \tilde{\lambda}^2(\frac{\omega}{v} + \mathrm{i}\alpha_{0d})^2 + 2\tilde{\mu}^2(k_z^{\mathrm{td}})^2 & 2\tilde{\mu}^2 k_z^{\mathrm{ts}} k_x \end{bmatrix}^{-1} \begin{bmatrix} -k_z^{\mathrm{inc}} \\ 0 \\ \tilde{\lambda}^1(\frac{\omega}{v} + \mathrm{i}\alpha_0)^2 \end{bmatrix}, \tag{20}$$

where the complex Lamé constants for a given k_x and k_z are given by

$$\tilde{\lambda}^1 = \frac{\rho\omega^2}{(k_x)^2 + (k_z^{\mathrm{inc}})^2}, \tag{21}$$

$$\tilde{\mu}^2 = \frac{\rho_s\omega^2}{(k_x)^2 + (k_z^{\mathrm{td}})^2}, \tag{22}$$

$$\tilde{\lambda}^2 = \frac{\rho_s\omega^2}{(k_x)^2 + (k_z^{\mathrm{inc}})^2} - 2\tilde{\mu}^2, \tag{23}$$

ρ being the density of the liquid and ρ_s the density of the solid.

The reflection coefficient is then RD and is found from (20). For each inhomogeneity and for each angle of incidence, we may calculate RD by simple matrix algebra.

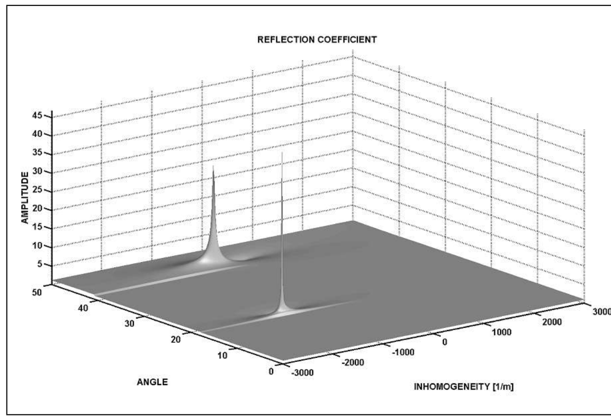

Figure 2. The amplitude of the reflected waves for different inhomogeneities and for different incidence angles on a brass/water interface. Note the appearance of two peaks. The one at an angle of 44.05 degrees is the Rayleigh peak, whereas the one at an angle of 19.01 degrees corresponds to the new kind of surface waves.

3. Numerical results

We neglect damping and consider a water/brass interface characterized by $\rho = 1000\,\mathrm{kg/m^3}$, $v = 1480\,\mathrm{m/s}$, $\rho_s = 8100\,\mathrm{kg/m^3}$, $v_s = 2270\,\mathrm{m/s}$ and $v_d = 4840\,\mathrm{m/s}$.

In Figure 2, we have plotted the absolute value of the reflection coefficient as a function of the inhomogeneity β^{inc} and the angle of incidence θ^{inc} for a frequency of 5 MHz. As expected, a peak can be found at the leaky Rayleigh wave angle. The exact values for that peak are: $\beta_R = -317.27/\mathrm{m}$ and $\theta_R = 44.05°$. However, for this water/brass interface, there appears another peak (as seen in Figure 2), for the values $\beta_D = -1269.39/\mathrm{m}$ and $\theta_D = 19.01°$. If Snell's law is applied to recover the corresponding velocity of both the leaky Rayleigh and the new phenomena, we obtain respectively $v_R = 2128.71\,\mathrm{m/s}$ and $v_D = 4544.17\,\mathrm{m/s}$. The latter differs form the former and also differs from the bulk velocities in the solid, whence it is not a leaky Rayleigh wave and not a lateral wave corresponding to a 'branch cut' [15, 16, 21]. If we examine the reflection coefficient for 1 MHz (instead of 5 MHz), we obtain $\beta_R = -63.42/\mathrm{m}$ and $\beta_D = -253.88/\mathrm{m}$ for exactly the same angles.

4. Properties of the new kind of surface waves

In what follows, we have taken a frequency of 5 MHz. We recall that the displacement is given by (15)–(16). We have normalized the displacements by dividing them by $N = |\omega/vRD|$. In other words the displacement is normalized in order to obtain a unit displacement value for the reflected sound in the liquid at the origin. This makes a comparison between Rayleigh waves and the new type of surface wave easier. In Figure 3, the profile of a leaky Rayleigh wave at 5 MHz is plotted, for an angle of incidence of 44.05 degrees and inhomogeneity −317.27/m.

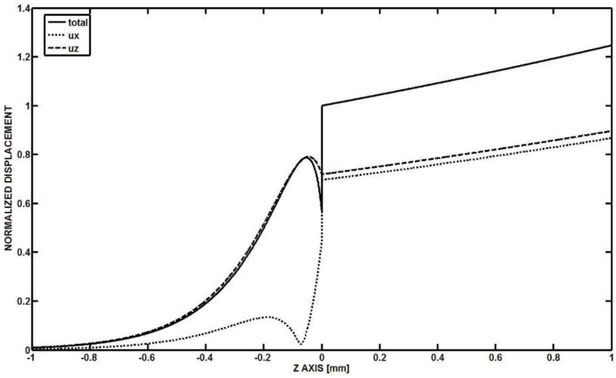

Figure 3. Normalized amplitude profile of a Rayleigh wave at 5 MHz. The angle of incidence is 44.05 degrees, with inhomogeneity −317.27/m. The dotted line corresponds to $|\boldsymbol{u}_x|$, the dashed line to $|\boldsymbol{u}_z|$, while the solid line corresponds to $|\boldsymbol{u}|$.

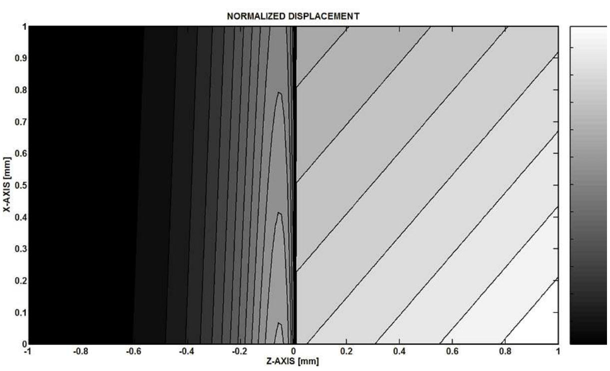

Figure 4. A filled contour plot of the normalized total displacement $|\boldsymbol{u}|$ of a Rayleigh wave (the one of Figure 3) as a function of z and the propagation distance x.

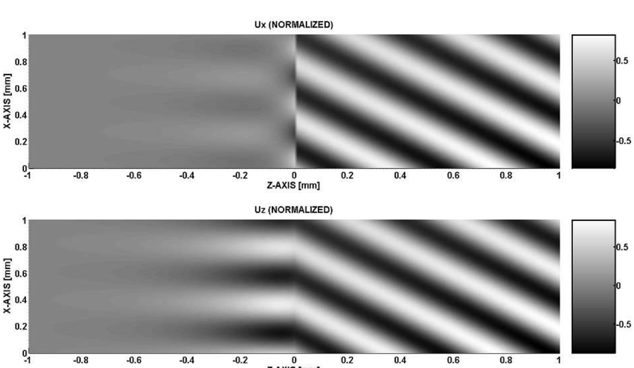

Figure 5. The real part of the normalized displacement (u_x: top; u_z: bottom) for time $t = 0$, as a function of z and the propagation distance x for the leaky Rayleigh wave of Figure 4. The leaky Rayleigh wave is elliptically polarized, which is a well known feature of such surface waves.

The propagation of this profile is depicted in Figure 4 and Figure 5, where we can see that the leaky Rayleigh wave is damped due to emission of sound into the liquid. In Figure 6, the particle displacement (in the solid side) at the spot $[x, z] = [0, 0]$ is given for each instant of time $t \in [0, 2\pi/\omega]$.

Figures 5–6 reveal that leaky Rayleigh waves are elliptically polarized.

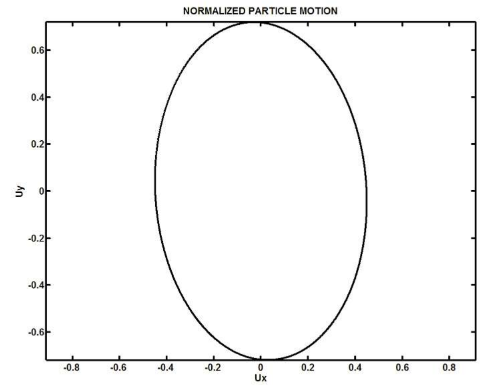

Figure 6. The normalized particle displacement in the solid for the leaky Rayleigh wave of Figure 5 at $[x, z] = [0, 0]$, depicted over 1 period of time. It is clear that this corresponds to elliptical polarization.

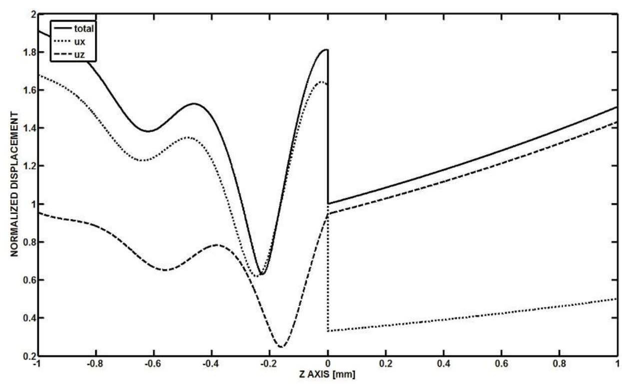

Figure 7. The normalized amplitude profile of the new kind of surface wave at 5 MHz The angle of incidence is 19.01 degrees, with inhomogeneity −1269.39/m. The dotted line corresponds to $|\boldsymbol{u}_x|$, the dashed line to $|\boldsymbol{u}_z|$, while the solid line corresponds to $|\boldsymbol{u}|$.

In Figure 7, the profile of the new kind of surface wave at 5 MHz, for an angle of incidence of 19.01 degrees and inhomogeneity −1269.38/m is shown. The new kind of surface wave consists of a lobe of increased amplitude near the interface that remains so while propagating (see Figure 8 and Figure 9), except for a decrease due to radiation into the liquid – as is the case for leaky Rayleigh waves. We will discuss this phenomenon further below. It is also noticed from Figure 10, that the polarization of the new kind of surface wave is also elliptical, though it differs from the polarization of a leaky Rayleigh wave (cfr. Figure 6).

It is widely known that a leaky Rayleigh wave consists of two inhomogeneous waves in the solid whose inhomogeneity vectors point into the liquid and one inhomogeneous wave in the liquid whose inhomogeneity vector also points into the liquid. Hence, radiation solely occurs in the direction of the liquid, i.e. the leaky Rayleigh wave leaks into the liquid. The exact wave vector components for the case of leaky Rayleigh waves are listed in Table I.

In the case of the new kind of surface waves (see Table I), radiation occurs both into the liquid as well as into the solid (through shear waves).

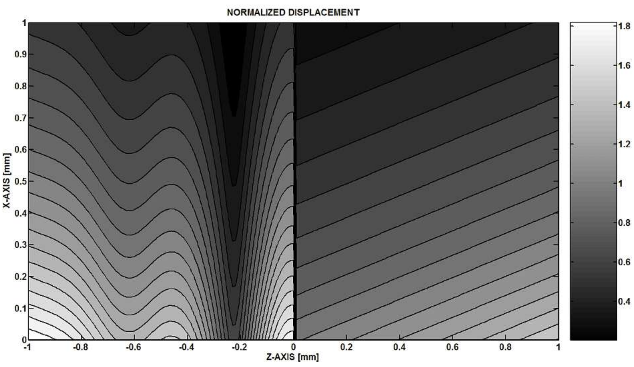

Figure 8. A filled contour plot of the normalized total displacement $|\boldsymbol{u}|$ of the new kind of surface wave (the one of Figure 7) as a function of z and the propagation distance x.

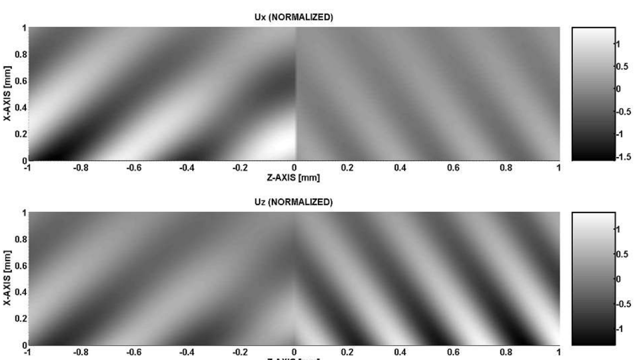

Figure 9. The real part of the normalized displacement (u_x: top; u_z: bottom) as a function of z and the propagation distance x for the new kind of surface wave of Figure 8. The new kind of surface wave is elliptically polarized (almost linearly).

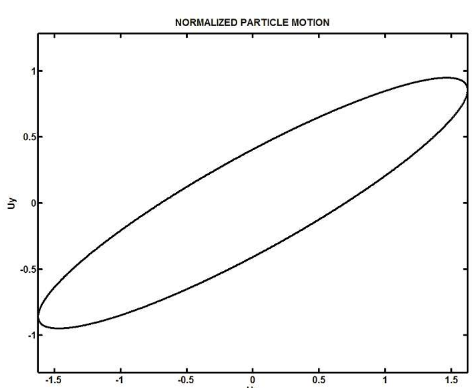

Figure 10. The particle normalized displacement in the solid for the new kind of surface wave of Figure 9 at $[x, z] = [0, 0]$, depicted over 1 period of time. Note that this corresponds to nearly linear elliptical polarization, and differs from the polarization of a leaky Rayleigh wave (see Figure 6).

5. Possibilities for the generation of the new kind of surface waves

Since bounded beams must be narrow enough for beam deformations (such as a Schoch-displacement) to occur when leaky Rayleigh waves are generated and since the incidence inhomogeneity for the generation of the new kind of surface waves is much larger than for the generation of a leaky Rayleigh wave, it is expected that the generation by and the effect on an incident bounded beam will be solely visible when it is much narrower than for the excitation of leaky Rayleigh waves. The drawback of applying such narrow beams is that they are very divergent and therefore only the nearfield might produce the expected effects; i.e. if a nice Gaussian near field can be fabricated. These complexities make the excitation of the new type of surface waves probably awkward and might be the explanation why the effects are not visible in reported experiments [10]

Table I. The calculated values (in 1/m) of the components of the wave vectors corresponding to the case of leaky Rayleigh waves and the new kind of surface waves, corresponding to the peaks that appear in Figure 2.

	Leaky Rayleigh wave	New kind of surface wave
k_x	$14759 + 228i$	$6926 + 1200i$
k_z^{inc}	$-15259 + 221i$	$-20105 + 413i$
k_z^{r}	$15259 - 221i$	$20105 - 413i$
k_z^{td}	$254 - 13256i$	$2530 - 3286i$
k_z^{ts}	$652 - 5166i$	$-12062 + 689i$

6. Conclusion

It is shown that the inhomogeneous wave theory predicts the existence and the excitation of a new kind of surface waves on a water/brass interface that radiates both into the solid and into the liquid. Its polarization is elliptical (almost linear). The importance of this new phenomena lies in the areas where leaky surface waves have found to be very important, such as geology, electronics and especially non destructive testing of materials. It is expected that the new kind of surface waves will be generated by bounded beams that must be much narrower than in the case of the generation of leaky Rayleigh waves. Difficulties concerning the fabrication of such narrow beams are also mentioned.

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