Available online at www.sciencedirect.com
ScienceDirect
Ultrasonics 44 (2006) e833–e837
Ultrasonics
www.elsevier.com/locate/ultras

Enhanced anisotropy in Paratellurite for inhomogeneous waves and its possible importance in the future development of acousto-optic devices

Nico F. Declercq [a,b,*], Nataliya V. Polikarpova [c], Vitaly B. Voloshinov [c], Oswald Leroy [d], Joris Degrieck [e]

[a] George W. Woodruff School of Mechanical Engineering, Georgia Institute of Technology, 801 Ferst Drive, Atlanta, GA 30332-0405, USA
[b] Georgia Tech Lorraine, 2 rue Marconi, 57070 Metz, France
[c] Department of Physics, M. V. Lomonosov Moscow State University, 119992 Moscow, Russia
[d] Interdisciplinary Research Center, Katholieke Universiteit Leuven, Campus Kortrijk, E. Sabbelaan 53, 8500 Kortrijk, Belgium
[e] Department of Mechanical Construction and Production, Ghent University, Sint Pietersnieuwstraat 41, B-9000 Gent, Belgium

Available online 6 June 2006

Abstract

The anisotropic feature of most crystals, involves a direction dependent wave velocity for each of the possible modes. Paratellurite (Tellurium dioxide) is extraordinary because, for one of the propagation modes, i.e. the quasi shear horizontal (QSH) mode, the anisotropy is exceptional. This results, on the one hand in a very strong directional dependent sound velocity and on the other hand, in a low wave velocity in certain directions, resulting in a high figure of merit for the acousto-optical interaction. In the case of inhomogeneous waves, the slowness surfaces change their shape and magnitude, for all crystals. However, for paratellurite, this effect is again extraordinary. As soon as a relatively small inhomogeneity is considered, the sound velocity for the QSH mode becomes really exceptionally anisotropic, resulting in a slowness surface that is almost spherical, covered by pins. The velocity corresponding to those 'pins', is much lower than in the case of homogeneous plane waves, which is very promising for the future development of acousto-optic cells involving an even higher figure of merit.

PACS: 43.20.Hq; 43.35.Sx; 43.20.−f; 43.20.Bi

Keywords: Paratellurite; Tellurium dioxide; Acousto-optics; Inhomogeneous waves; Slowness surface

1. Introduction

The new materials grown during the recent years are the crystals based on compounds of tellurium and mercury, e.g. paratellurite (TeO_2), calomel (Hg_2Cl_2), mercury bromide (Hg_2Br_2), mercury iodide (Hg_2I_2), etc. [1–3]. The single crystal of paratellurite (tellurium dioxide), applied in more than 90% of modern acousto-optic instruments, is one of the most efficient materials known so far in acousto-optics. Most acousto-optic devices operate in the Bragg regime and apply the acousto-optic effect for light deflection. The diffraction efficiency η in acousto-optics is the ratio of the intensity of the first order diffracted light and the zero order (un)diffracted light. If we consider the wavelength of light in free space, λ_0, the angle of light beam incidence (measured from the normal to the sound beam), θ, the length of interaction (the length of the light beam in the acousto-optic interaction area), L, the width of interaction (the length of the ultrasonic beam in the acousto-optic

* Corresponding author.
E-mail addresses: nico.declercq@me.gatech.edu, Declercq@ieee.org (N.F. Declercq).

0041-624X/$ - see front matter © 2006 Elsevier B.V. All rights reserved.
doi:10.1016/j.ultras.2006.05.113

interaction area), H, the average energy flow of acoustic power, P_a, then, in the Bragg regime of diffraction, this diffraction efficiency is given by [4–7]

$$\eta = \sin^2\left(\frac{\pi}{\lambda_0 \cos\theta}\left(\frac{M_2 L}{2H}P_a\right)^{1/2}\right) \tag{1}$$

with

$$M_2 = \frac{n_i^3 n_d^3 p^2}{\rho V^3} \tag{2}$$

for birefringent diffraction. The refractive index along the direction of incidence is given by n_i, whereas the one along the diffraction direction is n_d. The quantity M_2 is called the acousto-optic *figure of merit* and is completely determined by the properties of the considered media. The quantity p is the effective acousto-optic coefficient, and depends on the elasto-optic properties of the considered media.

The anisotropy of tellurium dioxide contributes to the extremely high acousto-optic figure of merit of the material $M_2 = 1200 \cdot 10^{-18}$ s^3/g and is about 1000 times higher than in the reference crystal of quartz [1,2]. The high figure of merit in paratellurite originates from the extremely low magnitude of the slow shear acoustic wave propagating along the [1 1 0] direction in the crystal ($V = 616$ m/s), only two times higher than the velocity of sound in air. On the other hand, the waves sent along the axes X ([1 0 0]) and Y ([0 1 0]) of the material, are characterized by the velocity value $V = 3050$ m/s, 5 times higher. Note that in moderate crystals such as quartz, the ratio of the maximal and minimal phase velocity values is less than 2.5 [8].

It may be expected that the elastic anisotropy influences not only the homogeneous bulk elastic waves propagation in paratellurite but also the propagation of inhomogeneous waves in the material [9,10].

2. Overview of classical slowness surfaces in paratellurite

The theoretical development for describing the propagation of ultrasound in piezoelectric crystals can be found in refs. [9,10].

Classically, taking into account Einstein's double suffix notation convention, the (real) wave vector $\mathbf{k}$ is replaced by $\mathbf{k} = l\omega(d_{1,g}\mathbf{e}_g)$ and is entered into the extended christoffel's equation for piezoelectric materials [9,10]. ω is the angular frequency. Then, for each (real) direction $(d_{1,x}, d_{1,y}, d_{1,z})$, the eigenvalue l can be determined. This l is then the slowness value. At the same time, the polarization vector $\mathbf{P}$ is determined as the eigenvector. We have developed a program that is able to draw 3D slowness surfaces and, when necessary, to add arrows that represent the polarization or the energy flow. For paratellurite, the slowness surfaces form a complicated 3D structure, consisting of three layers that intersect one another. The layers, i.e. the modes, separated according to their order of magnitude of the slowness for each direction, are then labeled according to the sound polarization and are named quasi longitudinal (QL), quasi shear horizontal (QSH) and quasi shear vertical (QSV). If it follows that the polarization is mainly directed along the propagation direction, the label QL is added. If it is mainly shear and directed along the XY-plane, the label QSH is added. If it is mainly shear and directed along the z-axis, the label QSV is added. The exact physical parameters of paratellurite, used in the presented results, can be found in Ref. [11].

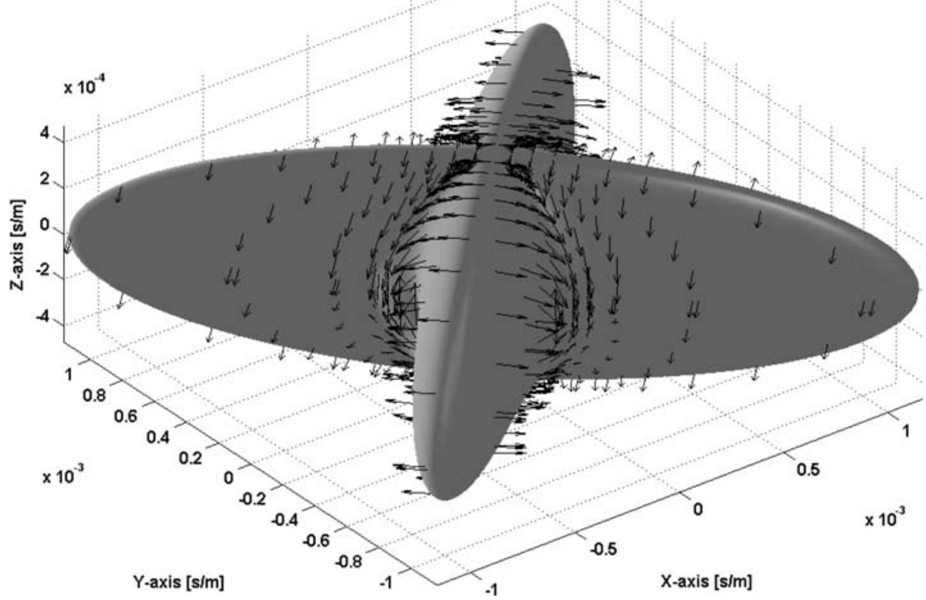

Fig. 1. Slowness surface of the QSH mode in paratellurite for homogeneous plane waves. The black arrows denote the polarization vector at each point on the slowness surface. Strong anisotropy is visible, resulting in four distinguishable lobes. This is a well known effect for this crystal.

In Fig. 1, the slowness surface for the QSH mode is depicted, together with black arrows that denote the polarization direction. Similar surfaces have been obtained for the QL and the QSV modes and they all agree with the cross-sections that have been presented in [11]. The surfaces can also be found in [12].

3. Inhomogeneous waves in Paratellurite

The behavior of homogeneous plane waves in paratellurite is well described in many references (e.g. ref [4]). Nevertheless, it is also known that inhomogeneous waves can exist in anisotropic media [9,10,13]. An inhomogeneous wave is defined as a plane wave having a complex wave vector $\mathbf{k}$. This results in an exponentially decaying amplitude along the wave front and in a phase velocity that differs from a pure plane wave. The notion of inhomogeneous waves, inside the bulk of a piezoelectric crystal, is introduced through the concept of a complex direction. For complex quantities, we attribute a subscript '1' to the real part and a subscript '2' to the imaginary part. A real direction is then defined as a real vector $\mathbf{d}_1$, for which $\mathbf{d}_1 \cdot \mathbf{d}_1 = 1$. This is generalized to a complex direction $\mathbf{d} = \mathbf{d}_1 + i\mathbf{d}_2$, for which $\mathbf{d} \cdot \mathbf{d}^+ = 1$. Then, it is possible to determine the value l from the following definition of the complex wave vector:

$$\mathbf{k} = \mathbf{k}_1 + i\mathbf{k}_2 = l\omega(\mathbf{d}_1 + i\mathbf{d}_2) \tag{3}$$

For every possible complex direction, it is possible to determine l. The number of combinations of $\mathbf{d}_1$ and $\mathbf{d}_2$ is reduced by introducing a complex direction that, for simplicity, contains no imaginary part along the z-axis. The theoretical expressions for these directions and also the resulting amplitudes of the wave vector for each possible Cartesian component, real and imaginary, can also be found in [9,10] where it has been shown that we can introduce an imaginarity parameter b that is a measure for the inhomogeneity of the considered inhomogeneous waves. We apply the same parameter b here.

In what follows, the slowness surfaces correspond, for each direction, to the magnitude of the real part of the slowness vector $\mathbf{s} = \mathbf{k}/\omega$.

3.1. Inhomogeneous QL modes

Fig. 2 shows the situation when b is 84/140. This figure can be compared with the cross-sections in [11] or the surfaces in [12]. It is seen that the slowness surface changes significantly and that this change depends on the direction. Nevertheless, the values of the radius of the slowness surface, remains of the same order of magnitude and it is therefore mainly the shape of the surface that changes. This means that, for this mode, the use of inhomogeneous waves results in different characteristics, though spectacular changes are not observed. For demonstration purposes, in Fig. 2, the black arrows denote the (real part of the) polarization vectors.

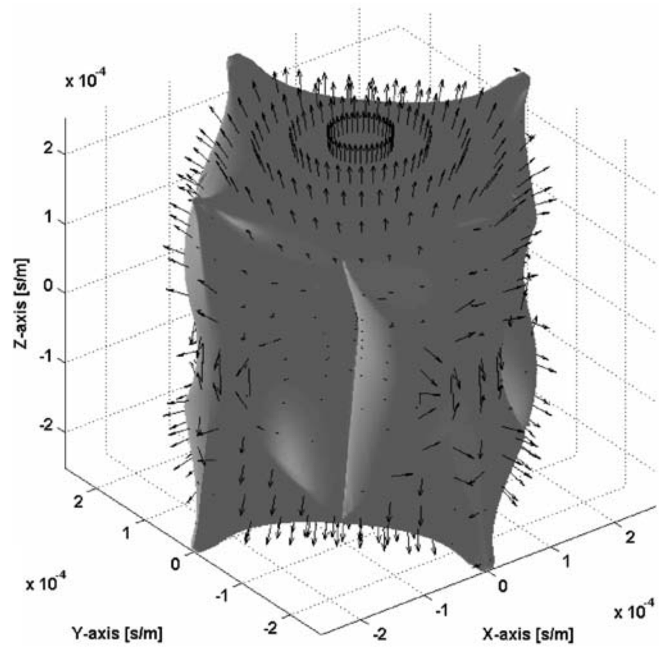

Fig. 2. Slowness surface of the QL mode in paratellurite for an inhomogeneity parameter $b = 84/140$. The black arrows denote the polarization vector at each point on the slowness surface. A significant deformation is visible compared to the situation where $b = 0$.

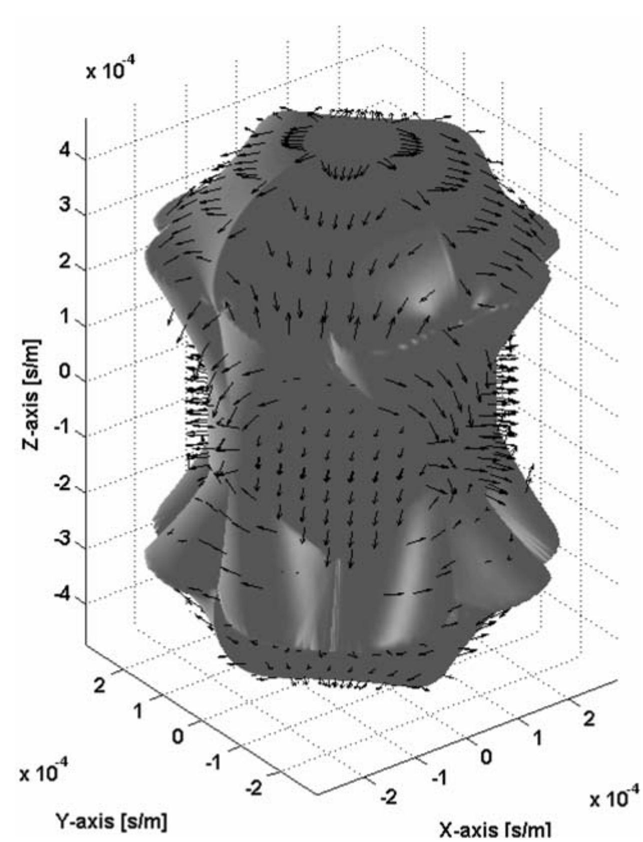

Fig. 3. Slowness surface of the QSV mode in paratellurite for an inhomogeneity parameter $b = 84/140$. The black arrows denote the polarization vector at each point on the slowness surface. A significant deformation is visible compared to the situation where $b = 0$.

3.2. Inhomogeneous QSV modes

Fig. 3 shows the situation when b is 84/140. This figure again can be compared with the cross-sections in [11] or the surfaces in [12]. It is again seen that the slowness surface changed significantly and that the change depends on the direction. Besides the change of shape, the magnitude is also changes considerably, whence the use of inhomogeneous waves, for this mode, will differ noticeably from the case of homogeneous plane waves. For demonstration purposes, in Fig. 3, the black arrows denote the (real part of the) polarization vectors.

3.3. Inhomogeneous QSH modes

Because we have noticed that something spectacularly happens to the QSH mode, when the inhomogeneity parameter b is increased, we first present the result for a small value of b, i.e. $b = 12/140$, in Fig. 4. Comparison of Fig. 4 with Fig. 1 shows that the 'lobes' in this slowness surface, are more outspoken in Fig. 4 than in Fig. 1. This means that the anisotropy has increased. Further growth of the inhomogeneity parameter b shows that a spectacular deformation of the lobes occurs, reflecting a spectacular increase of the anisotropy. The case of $b = 84/140$ is given in Fig. 5, where it is seen that the slowness curve has become almost like a sphere, covered by pins. This means that the slowness is almost isotropic for most directions, and becomes extraordinary large (corresponding to an extraordinary low propagation velocity), for certain directions. For demonstration purposes, the black arrows denote the polarization vectors. This effect of extraordinary anisotropy occurs in any crystal, but only for very large values of b. The fact that the effect occurs for paratellurite even for small values such as $b = 24/140$, is unique.

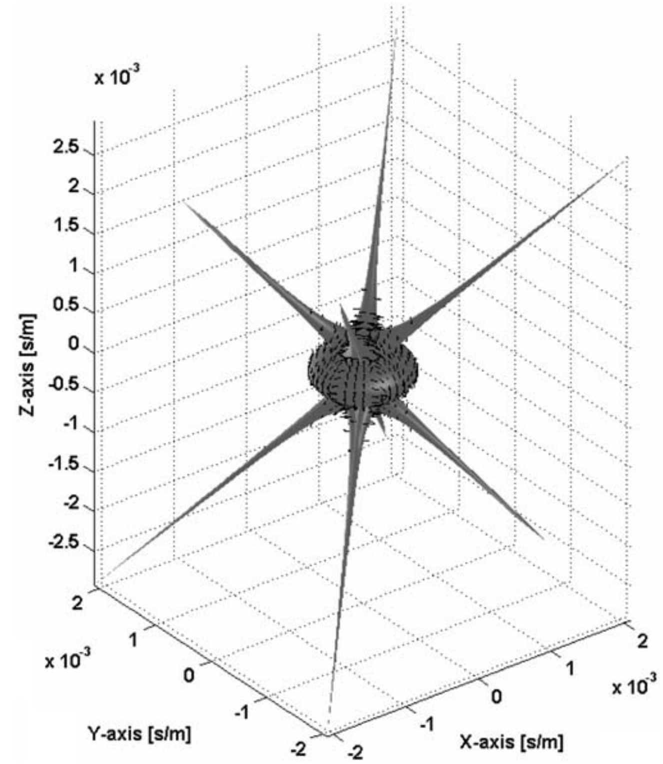

Fig. 5. Slowness surface of the QSH mode in paratellurite for an inhomogeneity parameter $b = 84/140$. The black arrows denote the polarization vector at each point on the slowness surface. Compared to Fig. 4, the anisotropy is further increased resulting in an almost sphere, covered by 8 pins.

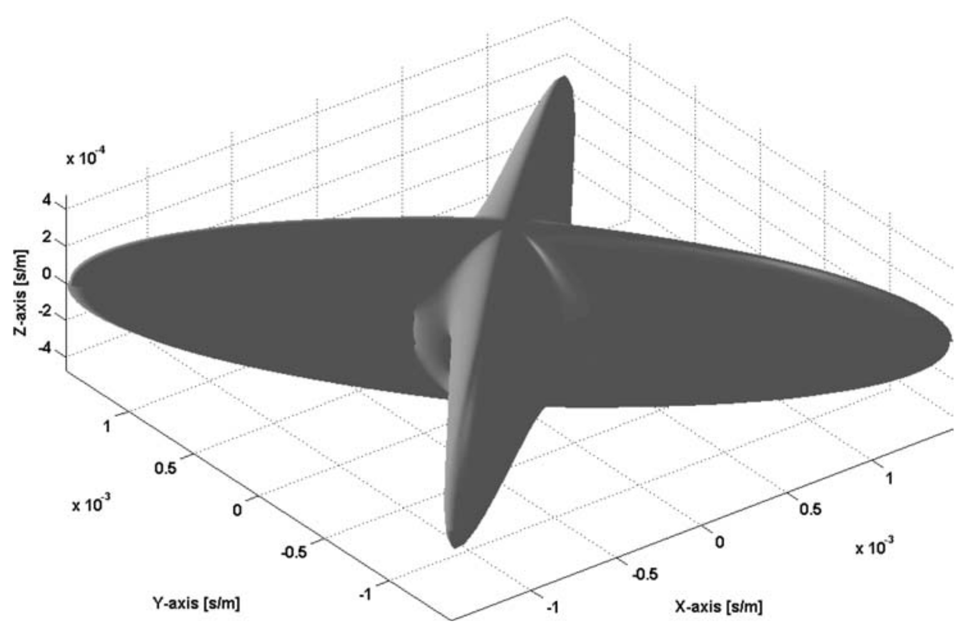

Fig. 4. Slowness surface of the QSH mode in paratellurite for an inhomogeneity parameter $b = 12/140$. Compared to Fig. 1, a slightly increased anisotropy is visible.

4. Possible consequences in acousto-optics

Acousto-optic devices could be manufactured in the future that produce bounded inhomogeneous bulk waves [14]. We would not be surprised that, especially in paratellurite, very interesting effects would be observable. This could generate new and modern acousto-optic devices.

5. Conclusions and prospects

In this paper, the description and the laws of propagation of bulk inhomogeneous plane waves in the TeO_2 single crystal, are examined. The results of the investigation indicate that the efficiency of TeO_2 for the acousto-optic interaction, may be further enhanced, by using inhomogeneous waves instead of homogeneous plane waves. This is due to the increased velocity difference for certain directions, compared to the case when only homogeneous plane waves are taken into account. Acousto-optic modulators, deflectors and filters should be mentioned in this context. Furthermore, it must also be mentioned that the peculiarities of the inhomogeneous waves behavior are typical not only for the paratellurite single crystals but for many other materials, if they are characterized by a very strong elastic anisotropy. For example, the results of the analysis may be generalized to the entire family of crystals such as the mercury halides (calomel, mercury bromide and mercury iodide). At the moment, these materials, as well as paratellurite, are very promising for the application in modern acousto-optic devices.

The numerical study in this report shows that it is possible to predict the existence of new acoustic and acousto-optics phenomena in crystals such a paratellurite. The investigation of laws and regular trends of the propagation of waves in the new materials may be the basic direction of the future scientific research. Therefore, the study is not only interesting from a fundamental point of view, but also from the point of applications.

References

[1] A. Yariv, P. Yeh, Optical Waves in Crystals, Wiley and Sons, New York, USA, 1984.

[2] J. Xu, R. Stroud, Acousto-Optic Devices, John Wiley & Sons, New York, USA, 1992.

[3] M. Gottlieb, A. Goutzoulis, N. Singh, High-performance acousto-optic materials: Hg_2Cl_2 and $PbBr_2$, Opt. Eng. 31 (N 10) (1992) 2110–2117.

[4] Jieping Xu, Robert Stroud, Acousto-Optic Devices: Principles Design and Applications, John Wiley & Sons, Inc, New York, 1992.

[5] G.A. Coquin, D.A. Pinnow, A.W. Warner, Physical properties of lead molybdate relevant to acousto-optic device applications, J. Appl. Phys. 42 (1971) 2162–2168.

[6] N. Uchida, Y. Ohmachi, Elastic and photoelastic properties of TeO_2 single crystal, J. Appl. Phys 40 (1969) 4692–4695.

[7] T. Yano, A. Watanabe, Acousto-optic figure of merit of TeO_2 for circularly polarized light, J. Appl. Phys. 45 (1974) 1243–1245.

[8] N.V. Polikarpova, V.B. Voloshinov, Application of acousto-optic interactions in anisotropic media for control of light radiation, Acustica-Acta Acustica 89 (2003) 930–935.

[9] Nico F. Declercq, Joris Degrieck, Oswald Leroy, Sound in biased piezoelectric materials of general anisotropy, Annalen der Physik (Ann. Phys. - Berlin).

[10] Nico F. Declercq, Joris Degrieck, Oswald Leroy, Inhomogenous waves in piezoelectric crystals, Acta Acustica United with Acustica 91 (2005) 840–845.

[11] B.A. Auld, Acoustic fields and waves in solids, second ed., Krieger, Florida, 1989.

[12] Nico F. Declercq, The interaction of complex harmonic elastic waves with periodically corrugated surfaces and with anisotropic viscoelastic and/or piezoelectric layered media, Ph.D dissertation, Ghent University, Faculty of Engineering, Department of Mechanical Construction and Production, 2005.

[13] Nico F. Declercq, Rudy Briers, Joris Degrieck, Oswald Leroy, The history and properties of ultrasonic inhomogeneous waves, IEEE-UFFC 52 (5) (2005) 776–791.

[14] Nico F. Declercq, Joris Degrieck, Oswald Leroy, The Laplace transform to describe bounded inhomogeneous waves, J. Acoust. Soc. Am. 116 (1) (2004) 51–60.