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Fringed sound fields and their interaction with liquid–solid interfaces

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Abstract

On the one hand, it is well known that Gaussian beams posses the ability to stimulate Rayleigh waves, resulting in the Schoch effect, a lateral beam displacement. This effect, often characterized by a reflected sound pattern consisting of two anti-phase beams, is due to the re-radiation of sound because of the stimulation of leaky Rayleigh waves. On the other hand, fringed sound beams are characterized by the fact that they consist of a number of neighboring anti-phase narrow beams. They are a first approximation of a sound field originating from a phased array of harmonic vibrating crystals in which each crystal vibrates in anti-phase compared to its neighbor. The individual lobes within the fringed sound pattern diverge much less than standard Gaussian beams of the same size. The current study investigates the interaction of fringed beams with a liquid–solid interface. It is found that under certain conditions, a fringed beam, incident at the Rayleigh angle, produces a reflected sound pattern that contains a wide lobe that is not fringed. It is also shown that under other conditions, contrary to the famous forward displacement of the reflected sound for incident Gaussian beams, a strong backward displacement occurs for fringed beams.

Keywords: Scattering; Bounded beams; Fringed beams; Lamb waves; Rayleigh waves

1. Introduction

In optics, the Goos–Hänchen theory predicts a lateral displacement of a light beam that is internally reflected from a dielectric interface [1]. This phenomenon appears when a bounded beam is incident from an optically denser medium at an angle close to the critical angle, resulting in the transfer of a portion of the energy into the rarer medium by means of the excitation of an electromagnetic field that travels along the interface. This energy leaks back into the denser medium and becomes part of the reflected beam exhibiting a lateral displacement that appears as a forward beam shift. This was studied by Tamir and Bertoni [2]. The early experiments of Schoch [3–5] applying the acoustic analog of the Goos–Hänchen effect for an acoustic beam reflected from a liquid–solid interface showed a forward lateral displacement of the reflected ultrasonic beam. It has also been shown before [6–10] that a backward displacement of an ultrasonic beam is possible on rough surfaces, when backward propagating surface waves are stimulated. The Schoch effect, which is, in the numerical examples in this paper accompanied by two reflected beams with a null strip in between, occurs when harmonic ultrasonic bounded beams are incident on a smooth liquid–solid interface at the Rayleigh angle. The expression 'null strip' was first mentioned in Neubauer and Dragonet [11].

Recently, a double sided bounded beam displacement has also been found under very particular circumstances of sound incident on a smooth glass plate [12]. Nevertheless, all these effects occur and are studied for incident Gaussian beams. The purpose of this paper is to examine the behavior

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of fringed bounded beams upon reflection on a smooth liquid–solid interface. Fringed bounded beams are a first approximation of the sound field generated by a phased array if each individual vibrates in anti-phase compared to its neighbor(s).

2. Description of 'fringed beams'

A fringed beam, propagating along the z-direction, is described in space as $F(x,z)$, with profile at $z=0$ given by

$$F(x,0)=\exp\left(-\left(\frac{x}{V}\right)^{64}\right)\sin\left(\frac{2\pi}{\Lambda}x\right) \quad (1)$$

The second factor on the right side of (1) causes the anti-phase state of subsequent lobes, whereas the first factor causes the fringed beam to be bounded in space. The number 64 in Eq. (1) is rather arbitrary and is chosen to obtain a sharp envelope. The periodicity Λ of the fringing pattern is given by

$$\Lambda=\frac{4V}{P}\frac{31}{30} \quad (2)$$

with P the number of lobes inside the beam. Because a fringed beam is a first approximation of the sound field originating from a phased array in which each subsequent element vibrates in anti-phase, the number P resembles the number of elements within a synthetic array transducer.

The behavior of fringed beams upon reflection on a liquid/solid interface is compared with the behavior of a traditional gaussian beam. The latter are described by $G(x,z)$, with profile.

$$G(x,0)=\exp\left(-\left(\frac{x}{W}\right)^{2}\right) \quad (3)$$

Because we are dealing with two entirely different beam patterns, it is necessary to make sure that, besides their equal amplitude, the amount of energy is equal as well. In other words:

$$\int_{-\infty}^{+\infty}G^2\,\mathrm{d}x=\int_{-\infty}^{+\infty}F^2\,\mathrm{d}x\simeq\int_{-V}^{+V}\sin^2\left(\frac{2\pi}{\Lambda}x\right)\mathrm{d}x \quad (4)$$

It can then be found straightforwardly that V is related to W and P as

$$V=\left|-\frac{15}{2}\frac{\sqrt{2}\pi^{3/2}WP}{124\sin^3(L)\cos(L)-62\sin(L)\cos(L)+15\pi P}\right|;$$
$$L=\frac{15}{62}\pi P \quad (5)$$

In the calculations that follow, we compare results for fringed beams (1) and Gaussian beams (3), and we have taken into account relations (2) and (5).

3. Beam reflection upon a liquid/solid interface

We have considered a water/brass interface. The density of water is 1000 kg/m^3, whereas the sound velocity is 1480 m/s. The density of brass is 8100 kg/m^3, whereas the longitudinal wave velocity in brass is 4840 m/s and the shear wave velocity is 2270 m/s. We have considered a Gaussian width $W=5$ mm. Results are given for 1 MHz and 5 MHz. The considered beams have been decomposed into homogeneous harmonic plane waves by means of the fast Fourier transform (FFT) along the X-axis. The FFT has been taken on the spatial interval $[-30W,+30W]$ on equally distributed sample spots. The number of sample spots and hence the number of plane waves in the plane wave decomposition is 2^N, N being the smallest positive integer for which

$$2^N\geqslant 3240\frac{W}{\Lambda} \quad (6)$$

The rigid (and overestimated) requirements (on the interval and on the number of samples) assures accurate simulations. The propagation of the considered beam is simulated by taking into account propagation of each of the plane waves in the Fourier decomposition. The latter is done by bearing in mind the dispersion relation for individual harmonic plane waves.

The reflection of the bounded beams is simulated by multiplication of each plane wave by its reflection coefficient and by taking into account propagation down to and up from the interface. Each reflection coefficient is obtained by taking into account continuity of normal stress and normal strain along the interface.

We did not see any significant phenomena at angles of incidence different from the Rayleigh angle. The latter is equal to $\Theta_R=44.05°$ for brass. Therefore only results are presented for the Rayleigh angle of incidence.

Fig. 1 presents the considered configuration. The angle of incidence is Θ, whereas the distance to the interface from the spot where the incident profile is considered, is equal to d. We always consider the reflected profile at an equal distance d in the direction of reflection. Results have been obtained for $d=0$, which artificially neglects diffraction effects due to propagation, and $d=W$, which corresponds better to reality and incorporates diffraction effects due to propagation.

In Fig. 2, the results for a Gaussian profile (dotted line) and a fringed beam (solid line) ($P=2$) at 5 MHz and for $d=0$, are shown. The reflected Gaussian beam consists of two lobes – this is the famous Schoch effect. The

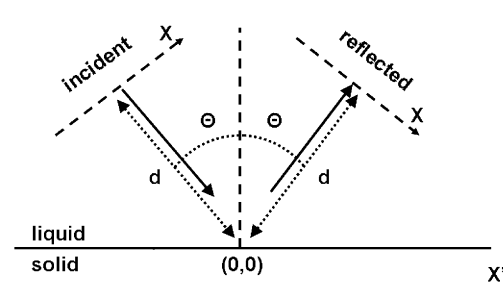

Fig. 1. A schematic of the configuration. Incident and reflected sound profiles are studied at a distance d from the interface, for an angle of incidence Θ.

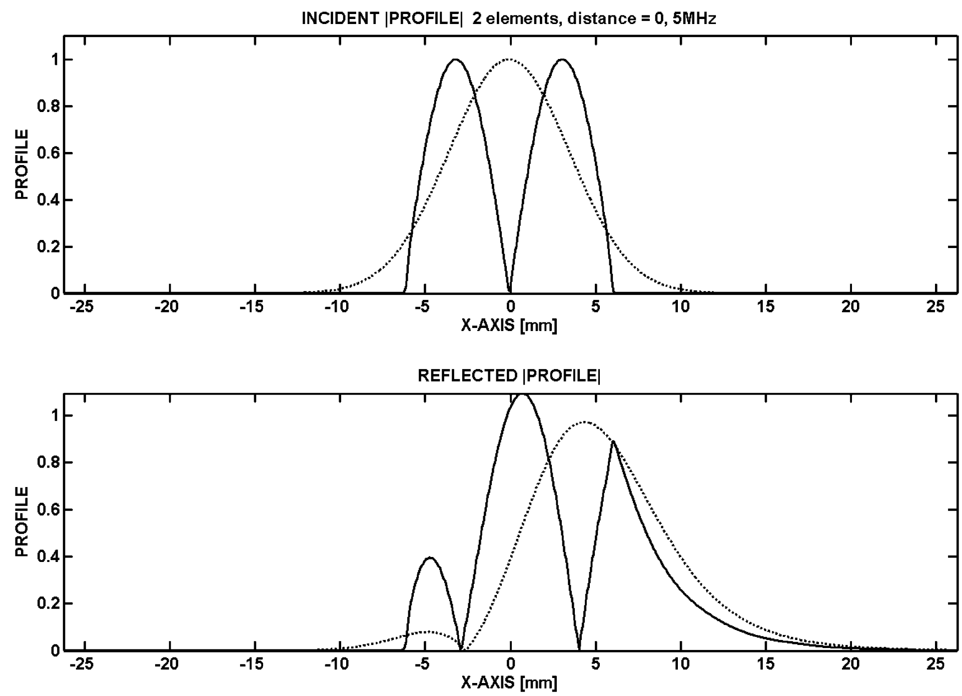

Fig. 2. Results for a Gaussian profile (dotted line) and a fringed beam (solid line) ($P = 2$) at 5 MHz and for $d = 0$. The reflected Gaussian beam consists of two lobes – this is the famous Schoch effect. The reflected fringed beam possesses one more (and wider) lobe than the incident number of lobes.

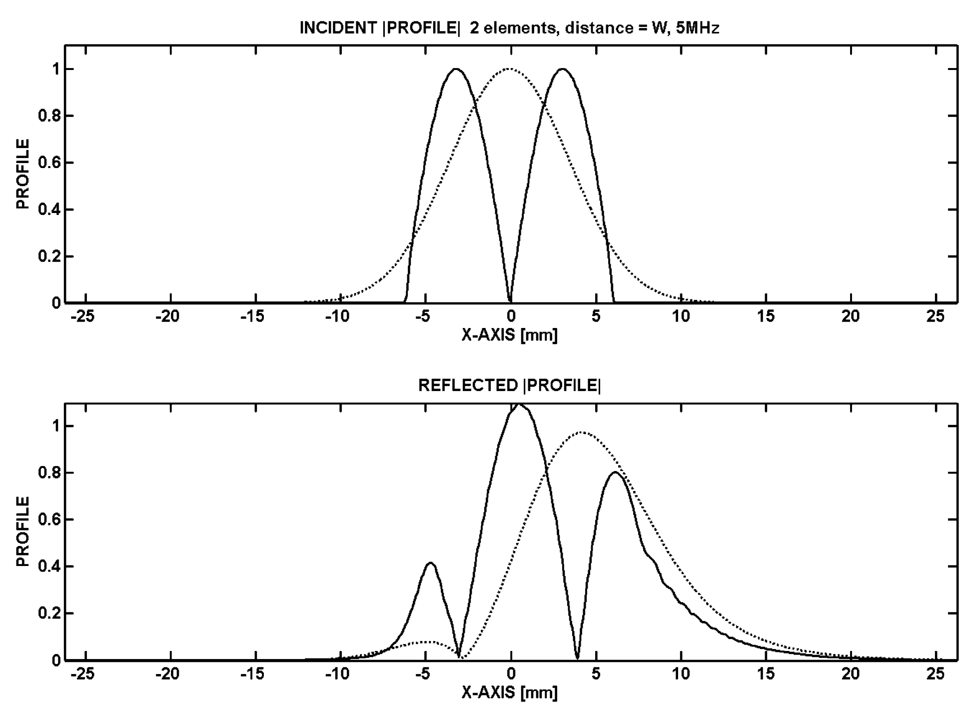

Fig. 3. The same result as in Fig. 2, but for a distance $d = W$. Besides the overall effect, additional 'irregularities' are visible due to diffraction effects along the propagation path.

reflected fringed beam possesses one more (and wider) lobe than the incident number of lobes. Just as for the case of Gaussian beams, the outer right lobe is due to re-radiating Rayleigh waves. Contrary to the case of Gaussian beams, the right lobe has a shape that is entirely different from the incident lobes. In order to study the effect of diffraction due to propagation, the result for $d = W$ is given in Fig. 3. It can be seen that, besides the overall effect, additional 'irregularities' are visible due those diffraction effects.

In Fig. 4, results are shown for a Gaussian profile (dotted line) and a fringed beam (solid line) ($P = 12$) at 1 MHz and for $d = 0$. Again, the reflected Gaussian beam consists

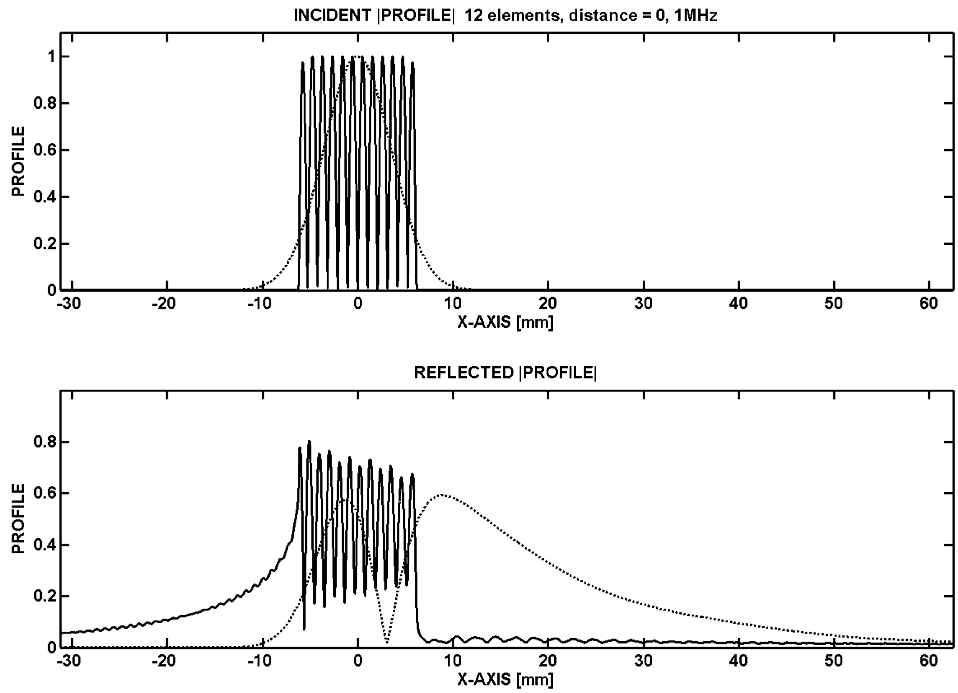

Fig. 4. Results for a Gaussian profile (dotted line) and a fringed beam (solid line) ($P = 12$) at 1 MHz and for $d = 0$. The reflected Gaussian beam consists of two lobes – this is the famous Schoch effect. The reflected fringed beam is considerably backward displaced, whereas almost no sound is forward displaced. This behavior is completely different compared with the behavior of Gaussian beams upon reflection on smooth interfaces.

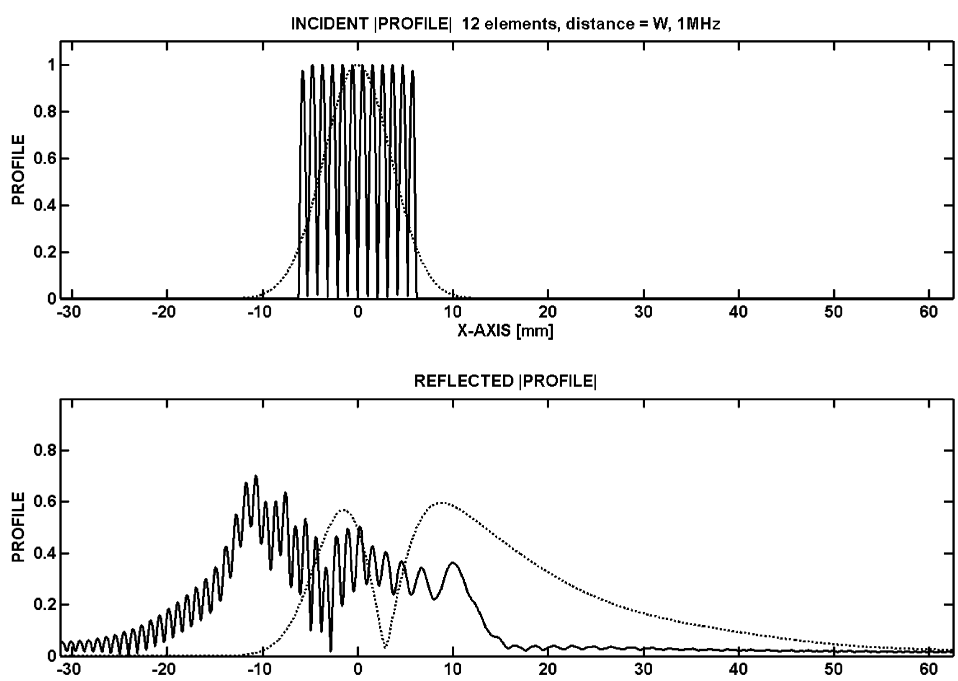

Fig. 5. The same result as in Fig. 4, but for a distance $d = W$. Besides the overall effect, additional 'irregularities' are visible due to diffraction effects along the propagation path. These effects cause sound to be displaced slightly in the forward direction as well. Most sound is still backward displaced.

of two lobes. The reflected fringed beam is considerably backward displaced, whereas almost no sound is forward displaced. This behavior is completely different compared with the behavior of Gaussian beams upon reflection on smooth interfaces.

The influence of diffraction effects due to propagation ($d = W$) is shown in Fig. 5. Note that the presence of diffraction effects eliminates the absence of sound in the forward direction. Still, most of the reflected sound is significantly backward displaced, which is extraordinary if

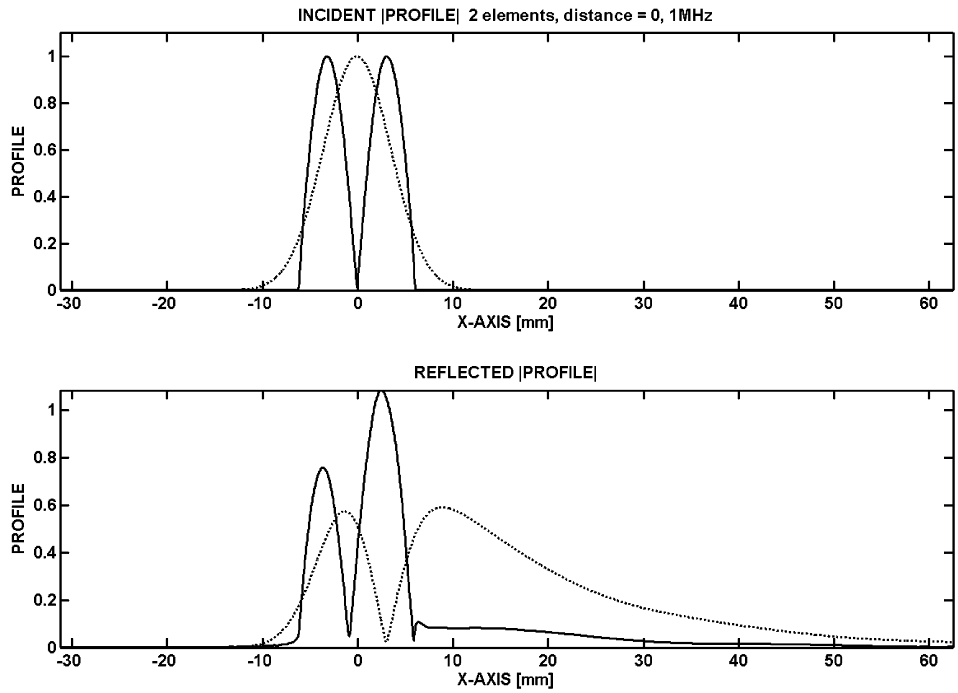

Fig. 6. The same result as in Fig. 2, but for 1 MHz. The additionally reflected lobe (situated in the forward direction compared with the other lobes) is almost like a plane wave.

compared with reflection effects for bounded Gaussian beams.

In some situations, it happens that the forward reflected beam is almost like a plane wave. This is illustrated in Fig. 6, where the result is shown for $P = 2$, $d = 0$ and a frequency of 1 MHz.

This effect becomes a little distorted in Fig. 7, where the propagation effect is also taken into account ($d = W$), but

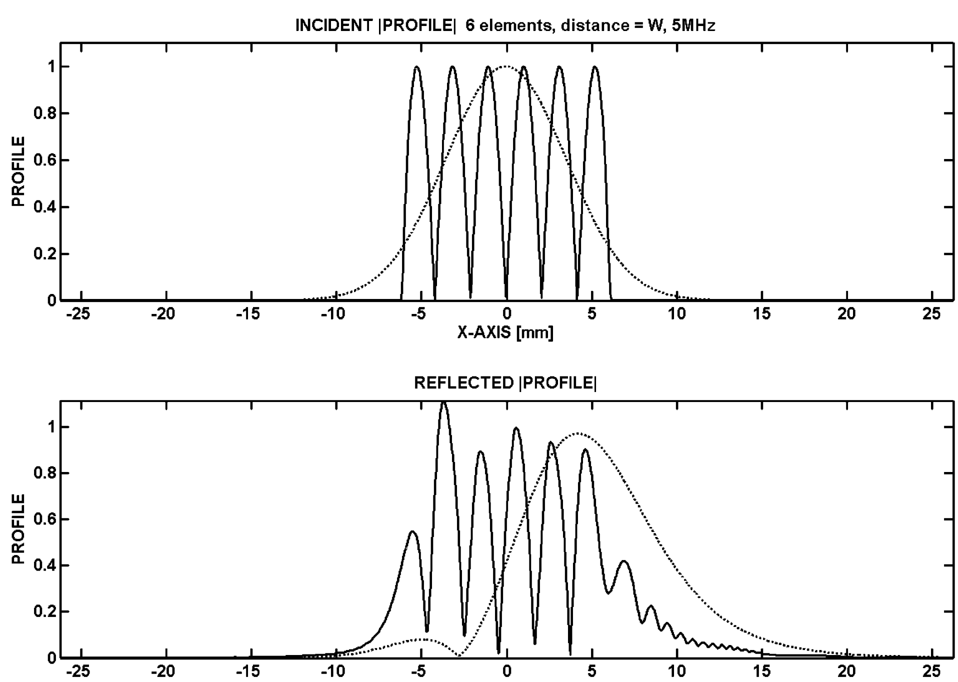

Fig. 7. The same result as in Fig. 6, but for a distance $d = W$. Besides the overall effect, additional 'irregularities' are visible due to diffraction effects along the propagation path. These effects make the forward shifted lobe less 'plane wave like'. It is still a wide lobe.

still, the forward reflected lobe is very wide compared with the incident lobes.

4. Conclusions

The reflection of fringed bounded ultrasonic beams has been studied and has been compared with Gaussian beams having the same amplitude and the same amount of energy. It is shown that the behavior of both types of waves is very different, especially at the Rayleigh angle of incidence. Under certain circumstances, extraordinary effects occur, such as a very significant backward displacement of the reflected beam. Sometimes it also happens that the reflected beam contains a forward displaced lobe that seems almost like a homogeneous plane wave. More scientific research is necessary to study the possibility to apply fringed sound beams in nondestructive testing of materials. Nevertheless, the presence of the listed extraordinary phenomena makes the consideration of fringed beams worthwhile during future research.

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