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On the theoretical possibility to apply an acoustic diffraction grating as a complex frequency filter device for electronic signals

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Abstract

In electronics, it is well known that filtering devices can be made that are able to decompose a signal instantaneously into a number of real frequency components. This procedure is equivalent to a numerical real time Fourier transform. However, it is also known that an electronic signal can be decomposed not just in real frequency components, but also in complex frequency components. The current paper shows that it is theoretically possible to create a device, made of a periodically rough surface and a system that transforms the electronic signal into acoustic waves, that can be used to measure the amplitude attributed to considered complex frequency components of an electronic signal, in real time. This 'thought device' is mainly based on the directivity of diffracted sound and the complex frequency dependence of this directivity.

Keywords: Complex harmonic waves; Diffraction; Transient waves; Inhomogeneous waves; Corrugated surfaces; Acousto-optics

1. Introduction

From an old paper of Spitzenogle and Quazi [1] it can be learned that a 'time limited signal' can be decomposed into a summation of signals with exponentially varying amplitude as a function of time. This principle has later been translated [2] to space limited acoustic signals (bounded beams) and appeared to be excellent in explaining what happens to sound at the Rayleigh angle of incidence. The latter formed a strong impetus for the development of the inhomogeneous wave theory, which explains the behavior of plane waves having complex parameters. Lately, special attention was drawn on the case of inhomogeneous waves having complex frequency [3–6]. Such complex harmonic plane waves therefore return the inhomogeneous wave theory back to the paper of Spitzenogle and Quazi [1]. In acoustics, complex harmonic waves are building blocks of sound beams, bounded in space and in time. In electronics, signals are often not really bounded in time. However one is frequently interested in the instant properties of such signals. For that reason, a numerical spectrogram can be obtained that gives the amplitude as a function of the real frequency and as a function of time. An experimental spectrogram can also be formed (with some errors) in real time by an electronic filter system. Nevertheless, it may likewise be important to consider a 'spectrogram' of a signal not just in the real frequency space, but also in the more general complex frequency space. This is mathematically reachable within a time limited window [1], but it must also be possible to build a device that performs such a spectrogram in real time somehow by means of a filter system. In what follows, it is shown from a theoretical point of view, that an acoustic diffraction grating is a possible tool for that purpose.

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2. Complex harmonic plane waves

A thorough description of complex harmonic plane waves can be found in [3–6]. Therefore, we make this discourse very short. Complex harmonic plane waves are described by a particle displacement vector $\mathbf{u}$ as follows:

$$\mathbf{u} = A\mathbf{P}\exp(\mathrm{i}\mathbf{k}\cdot\mathbf{r} - \mathrm{i}\omega t) \tag{1}$$

Besides time t and space $\mathbf{r}$, all parameters in (1), i.e. the amplitude A, the polarization $\mathbf{P}$, the wave vector $\mathbf{k}$ and the angular frequency ω can be complex valued. Complex harmonic plane waves differ from harmonic plane waves by an angular frequency ω that is complex valued instead of real valued. Hence,

$$\omega = \omega_1 + \mathrm{i}\omega_2; \quad \omega_1, \omega_2 \in \Re \tag{2}$$

and

$$\mathbf{u}(\mathbf{r} = 0, t) = A\mathbf{P}\exp(\omega_2 t)\exp\mathrm{i}(-\omega_1 t) \tag{3}$$

Henceforth, the term 'frequency' denotes $\omega/(2\pi)$ while the term angular frequency will be specifically used for ω itself. It is seen in (3) that the amplitude changes exponentially in time, through ω_2. For consistency with Ref. [1], we only consider $\omega_1 \geqslant 0$ and $\omega_2 \geqslant 0$. Complex harmonic waves are a solution of the visco-elastic wave equation if the dispersion relation holds [6]:

$$\mathbf{k}\cdot\mathbf{k} = \left(\frac{\omega}{v_0} - \mathrm{i}\alpha_0\right)^2 \tag{4}$$

with v_0 the phase velocity of plane waves having a real wave vector and real frequency and with α_0 the intrinsic damping of the considered media.

Important for what follows is the definition of the energy velocity vector $\mathbf{v}_\mathrm{E}$. It can be found in [3,4,6] that $\mathbf{v}_\mathrm{E}$ is proportional to

$$\frac{\mathbf{k}_2\omega_2 + \omega_1\mathbf{k}_1}{\omega_1^2 + \omega_2^2} \tag{5}$$

with $\mathbf{k}_1 = \mathrm{Re}(\mathbf{k})$ and $\mathbf{k}_2 = \mathrm{Im}(\mathbf{k})$.

3. The diffraction of complex harmonic plane waves

When harmonic plane waves, possessing a wave vector component k_x^{inc} along the interface, interact with a corrugated surface (see Fig. 1), having a periodicity Λ, they diffract into harmonic plane waves of different orders 'm'. Each of these waves have a wave vector component

$$k_x^m = k_x^{\mathrm{inc}} + m\frac{2\pi}{\Lambda} \tag{6}$$

along the interface and a component k_z^m perpendicular to the interface given by the dispersion relation (4). It has been shown before that this grating equation (6) is also valid for incident inhomogeneous waves. This is because

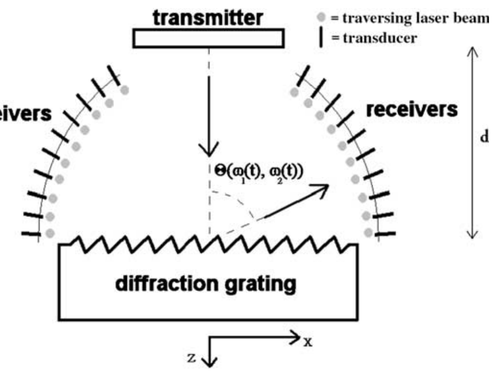

Fig. 1. A schematic of the 'thought device', based on sound normally incident on a diffraction grating and diffracted into several directions. A discrete number of omni-directional transducers is placed at constant distance and different angles. In front of each transducer there is a laser beam traversing the sound beam in order to obtain the phase propagation direction as distinguished from the energy propagation direction.

it represents the spatial periodicity of the phase along the periodic interface and not the amplitude. Because the imaginary part of the frequency, of a given incident sound wave, influences the amplitude and not the phase, it is evident that the grating equation (6) also holds for incident complex harmonic plane waves.

For a given diffraction order 'm' and its wave vector component k_x^m along the interface, the sign of the wave vector component k_z^m normal to the interface is chosen according to well known sign choice conventions [7–10] in terms of the phase propagation direction, based on causality principles, interpreted here in terms of the energy propagation direction.

4. How the acoustic diffraction grating may work

The dispersion relation (4) represents two real equations, whereas the propagation of a complex harmonic plane wave is characterized by two unknown vectors, $\mathbf{k}_1$ and $\mathbf{k}_2$, and two unknown scalars, ω_1 and ω_2. For a chosen Euclidian space, the grating equation and incident wave propagation direction determine one vector component (in 2D space) or two vector components (in 3D space) of both $\mathbf{k}_1$ and $\mathbf{k}_2$. This means that in order to be identified, there must be two independent parameters quantified, for example ω_1 and ω_2, or for example two independent quantities that contain unambiguous information about $\mathbf{k}_1$ and $\mathbf{k}_2$.

In what follows, we show numerically that, under the right conditions, the direction of the energy velocity vector $\mathbf{v}_\mathrm{E}$ as well as the direction of the phase velocity vector can be applied for that purpose, because for a given direction of the energy velocity vector $\mathbf{v}_\mathrm{E}$ as well as the direction of the phase velocity vector, a unique couple (ω_1, ω_2) exists.

Consider an electronic signal $s(t)$ that is decomposed into a series of complex frequency signals, i.e.

$$s(t) = \sum_n F_n(t) \exp(\mathrm{i}\Omega_n(t)t) \tag{7}$$

Within a time limited interval around a certain t_0, a representation is possible with constant $S_n = F_n(t_0)$ and $\omega_n = \Omega_n(t_0)$, whence

$$s(t)_{t\simeq t_0} = \sum_n S_n \exp \mathrm{i}\omega_n t \tag{8}$$

In the 'thought device' of Fig. 1, this signal can be transmitted to a transducer (emitter) perpendicularly directed to a diffraction grating at a distance d. If for a moment the diffraction grating is not considered, then a receiver at distance $2d$ will receive the acoustic signal and will transform it again into an electronic signal. This output signal will differ from the input signal because the system signal > transducer > propagation medium > transducer > signal has a transfer characteristic different from unity. Hence, the output signal S_{out} is related to the input signal S_{in} as follows:

$$S_{\mathrm{out}} = T(S_{\mathrm{in}}) \tag{9}$$

in which T is an operator who changes the amplitude attributed to each complex frequency component in the signal. If there is a grating and if the receiving transducer (see short bold lines in Fig. 1) is placed at a distance d from the surface at a certain direction, then there will also a transfer operator R be involved, which corresponds to the reflection coefficient for the received reflected order.

Therefore, the received signal is ultimately described by

$$s(t)_{t\simeq t_0} = \sum_n RTS_n \exp \mathrm{i}\omega_n t \tag{10}$$

As a consequence, if R and T are known for each possible ω_n, then measuring RTS_n reveals the values of S_n, i.e. the problem is reduced to measuring RTS_n as a function of each required ω_n. Therefore, the problem is reduced to defining what experimental 'condition' corresponds with each ω_n and measuring the accompanying amplitude for each of those 'conditions'. In what follows, it will be shown that measuring the phase propagation direction together with the energy propagation direction, defines a unique 'condition'. The energy propagation direction can practically be determined by a classical omni-directional transducer, whereas the phase propagation direction can be determined by the diffraction of laser light. The first is possible because an omni-directional transducer at a given spot measures by definition the encountered sound intensity, whereas the latter is possible because sound forms a diffraction grating for laser light and diffracts a laser light beam into different diffraction orders perpendicular to the sound wave fronts. This is the basic principle of acousto-optics. Both principles are depicted in Fig. 2, where gray circular areas denote a cross section of a (diffracted) laser beam and where again the bold line represents the receiver. The set of short parallel lines in Fig. 2 represent the wave fronts. The energy propagation direction and the phase propagation direction are denoted by a long arrow.

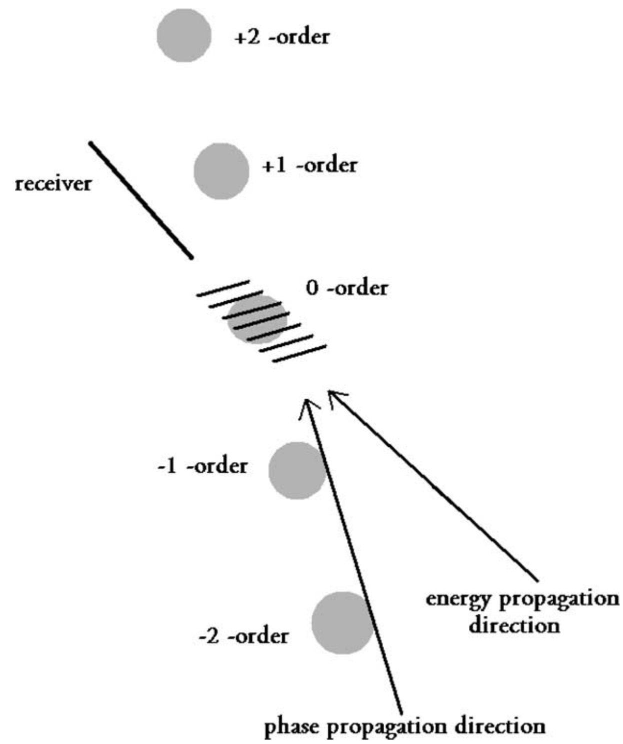

Fig. 2. A close up of the omni-directional transducers of Fig. 1, with attention to the diffraction direction of the traversing laser light. The difference between energy propagation direction and phase propagation must be measurable with this principle.

5. Numerical simulations

As explained, measuring the amplitude corresponding to a certain phase propagation direction and energy propagation direction, reveals the magnitude of a particular complex frequency component of a signal. There is however a limitation. When sound is received at a certain angle, it can only be attributed for certain to an order 'm' if there is no interference possible with other diffraction orders. Hence, a complex frequency spectrogram is solely possible within a certain interval in which there is no confusion possible between different diffraction orders. For that purpose the grating periodicity must be so that the maximum considered real frequency corresponds to the critical frequency for second order reflected waves. This critical frequency is defined as the largest frequency for which the second order reflected wave is evanescent (i.e. sticking to the interface). Practically this corresponds to the second order Scholte–Stoneley wave generating frequency, see for example Refs. [7,8]. The lowest frequency under consideration is then the critical frequency for first order waves, i.e. the first order Scholte–Stoneley wave generating frequency, because below this frequency, except for zero order reflected sound, there is no sound reflected from the interface.

We consider a periodically corrugated interface between water and a solid. The density of water is $1000\,\mathrm{kg/m^3}$, the plane wave velocity in water is 1480 m/s and the periodicity of the corrugation is $\Lambda = 250\,\mu\mathrm{m}$. For simplicity, we have only considered zero intrinsic damping. In Fig. 3, the calculated phase propagation direction (here equal to the energy propagation direction) is depicted for normal incident harmonic homogeneous plane waves having real frequencies between 4 MHz and 16 MHz. The dotted line corresponds to the first order reflected plane waves, whereas the solid line corresponds to second order reflected plane waves. The critical frequencies mentioned above, are directly visible as the frequencies at which the propagation direction starts to deviate from 90°, measured from the normal to the interface. Note that between approximately 6 MHz and 12 MHz there are only bulk reflected waves of the first order. This is the frequency region that we will focus on here in order to avoid interference between second and first order reflected sound waves. A 'thought device', as considered here, will therefore only be able to filter between a limited frequency interval, i.e. between 6 MHz and 12 MHz. Whenever necessary, this interval can be changed by using another grating periodicity.

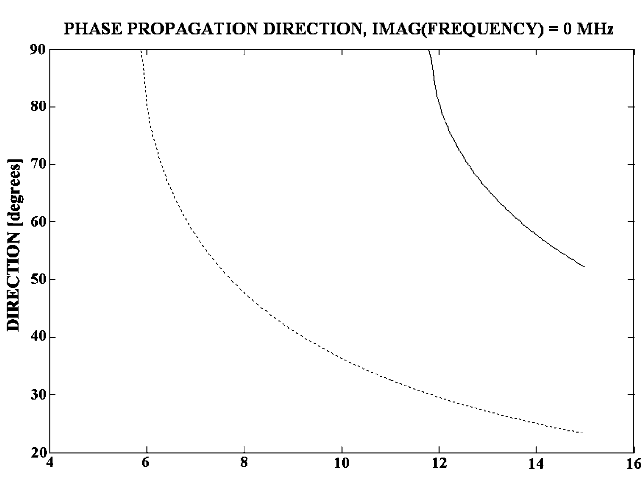

Fig. 3. The phase propagation direction for first and second order reflected sound coming from normal incident harmonic sound at different real frequencies. Dotted line: first order, solid line: second order.

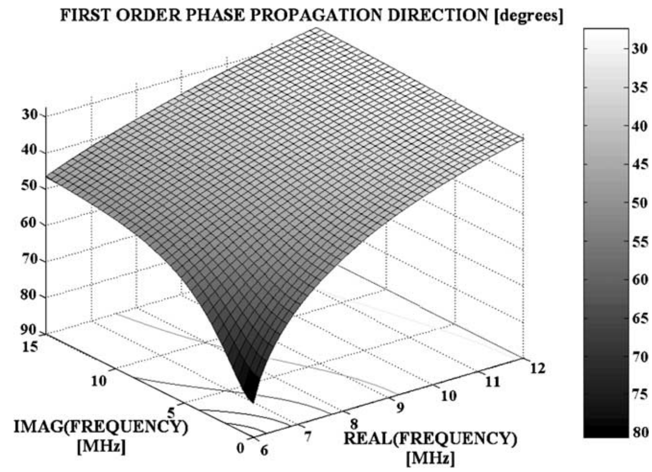

Fig. 4. The first order phase propagation direction as a function of the complex frequency for normal incident sound. The lines on the horizontal surface are projected equi-amplitude lines.

In Fig. 4 respectively Fig. 5, the phase propagation direction and energy propagation direction are depicted as a function of the real and imaginary frequency. Comparison of these figures reveals, that for every couple (ω_1, ω_2) within the considered interval, there is one energy propagation direction and there is one phase propagation direction. Furthermore, note that the projected equi-amplitude lines of one figure do not intersect more than once the projected lines in the other figure and therefore, for every combination of an energy propagation direction and a phase propagation direction, there corresponds only one couple (ω_1, ω_2). This shows

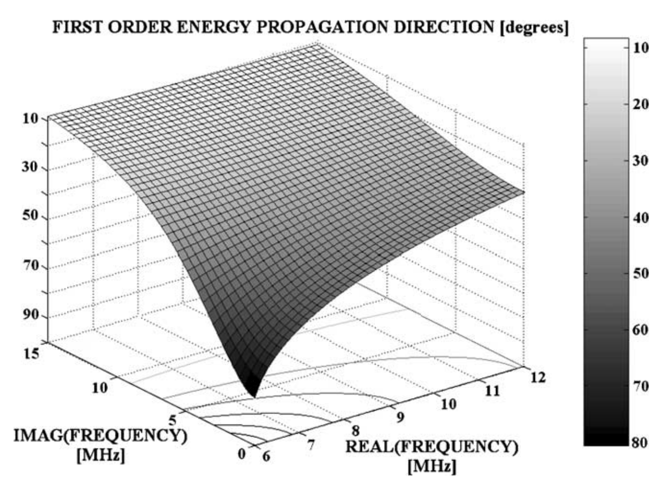

Fig. 5. The first order energy propagation direction as a function of the complex frequency for normal incident sound. The lines on the horizontal surface are projected equi-amplitude lines. Comparison of Fig. 4 reveals that for a given phase propagation direction and energy propagation direction, there corresponds only one couple (ω_1, ω_2). The reason is that each equi-amplitude line here does not cross a equi-amplitude line of Fig. 4 twice.

numerically that there is a 'one to one' relation between the measurable quantities and (ω_1, ω_2). In other words, if the phase propagation direction and the energy propagation direction are measured instantaneously, together with the amplitude of the received sound, the complex frequency and its attributed amplitude are found.

6. Conclusions

It is recalled that a time varying electronic signal can be decomposed not just into real frequencies, but also into complex frequencies. The first method can be performed electronically by means of a filtering device. The second is more difficult. Here we have theoretically proposed a method based on a diffraction grating together with the use of omni-directional transducers and the diffraction of laser light. The basic principle is the fact that for each complex frequency there is a unique couple of energy propagation and phase propagation direction.

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