A theoretical study of special acoustic effects caused by the staircase of the El Castillo pyramid at the Maya ruins of Chichen-Itza in Mexico

Nico F. Declercq[a)] and Joris Degrieck
Soete Laboratory, Department of Mechanical Construction and Production, Faculty of Engineering, Ghent University, Sint Pietersnieuwstraat 41, 9000 Gent, Belgium

Rudy Briers
KATHO (RENO dept), Sint Jozefstraat 1, 8820 Torhout, Belgium

Oswald Leroy
Interdisciplinary Research Center, KULeuven Campus Kortrijk, E. Sabbelaan 53, 8500 Kortrijk, Belgium

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It is known that a handclap in front of the stairs of the great pyramid of Chichen Itza produces a chirp echo which sounds more or less like the sound of a Quetzal bird. The present work describes precise diffraction simulations and attempts to answer the critical question what physical effects cause the formation of the chirp echo. Comparison is made with experimental results obtained from David Lubman. Numerical simulations show that the echo shows a strong dependence on the kind of incident sound. Simulations are performed for a (delta function like) pulse and also for a real handclap. The effect of reflections on the ground in front of the pyramid is also discussed. The present work also explains why an observer seated on the lowest step of the pyramid hears the sound of raindrops falling in a water filled bucket instead of footstep sounds when people, situated higher up the pyramid, climb the stairs. © *2004 Acoustical Society of America.*
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I. INTRODUCTION

During the post meeting tour of the first PanAmerican/Iberian meeting on Acoustics that was held in Cancun (Mexico) in 2002 (hereafter called ‘the post meeting tour’), the participants were shown that there are plenty of interesting sound effects that occur at Chichen-Itza. Chichen Itza is a Maya ruin where, besides the famous ‘‘ball court,’’[1] there is a pyramid (El Castillo) that produces a sound echo, in response to a handclap, which sounds like the chirp of a Quetzal bird. This effect has been one of the major subjects during plenty of talks given by David Lubman[2–5] and others.[6–8] Lubman has stressed the fact that the Quetzal bird chirp is actually caused by Bragg scattering. However, there has never been presented an actual simulation of the effect, except for some heuristic simulations based on the ray theory[2–5] or a heuristic approach for the case of incidence at 45° measured from the normal to the surface.[8–12] In what follows, a full diffraction simulation is presented of the echo, based on a (time-) delta function like handclap and also a real handclap, based on the physical parameters of the staircase of the Pyriamid at Chichen-Itza and based on the monofrequent single homogeneous plane wave diffraction theory of Claeys *et al.*,[9,10] which is a simplified case of the inhomogeneous plane wave diffraction theory.[11] The present work describes the first simulations of a spherical sound pulse, based on that monofrequent pure plane wave diffraction theory.[9,10] Furthermore, it is for the first time that the theory has been applied to audio frequencies.

Before presenting this development, it is of cultural importance to stress the fact that some people believe that the Quetzal bird chirp echo is caused by accident and others believe that it is caused as a consequence of the Pyramid builders’ purpose. Nevertheless, it is known that the Quetzal bird has played a very important role in Mayan culture, which is probably due to the fact that Mayans originally lived for many centuries in the forest before getting involved in the construction of cities and religious sites. However, what is sure about this pyramid is that it certainly functioned as a great solar calendar. For example a large serpent is built on one side that causes special light effects around the time of spring and fall equinox. This serpent is culturally connected to the Quetzal bird (as can be seen on a Mayan glyph from the Dresden Codex), whence the generation of a Quetzal bird echo might not be a real coincidence. It is also known that an echo in Mayan culture represents a spirit. However, it must also be notified that a Quetzal bird echo also occurs at other Pre-Columbian sites and Ancient Mexican ruins.[12] Furthermore the first author encountered similar effects as in Chichen Itza at two religious sites in Sri Lanka. There, the short concrete staircase, that enables people to take a bath in the Menik Ganga river at the religious site of Katharagama, produces the low frequency sound of quacking ducks in response to a handclap. Furthermore high frequency echoes occur on the immense staircase leading to the religious site of Sri Pada (Adam’s peak). Nevertheless, the ef-

[a)]Electronic mail: NicoF.Declercq@UGent.be

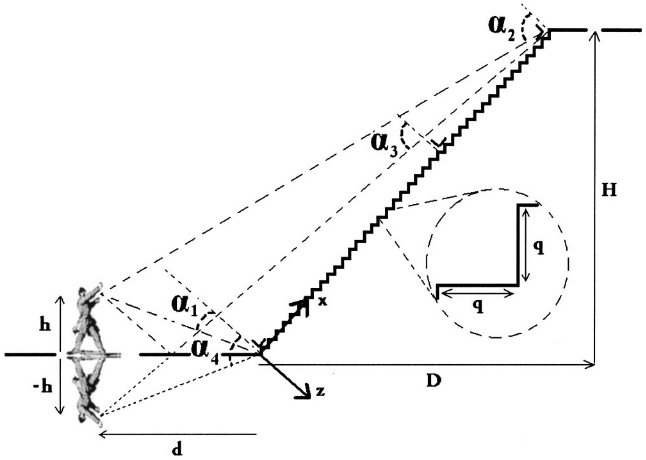

FIG. 1. Depiction of the pyramid's staircase with and observer in front of it.

fects in Sri Lanka are probably a coincidence and are not a result of purposely construction.

The last part of this paper is devoted to the less known fact that an observer seated on the lowest stair step of the great pyramid at Chichen Itza, hears pulses that sound like raindrops falling in a water filled bucket, when other people are climbing the pyramid higher up. This phenomenon (hereafter called ''raindrop effect''), has been observed by the first author and by a student fellow Cécile Goffaux during the post meeting tour. Since the ''rain god'' plays a very important role in the Yucatan Mayan culture, this finding might be an impetus for future cultural studies.

II. THEORETICAL DEVELOPMENT OF THE ECHO SIMULATION

The staircase is seen as a periodically corrugated (infinite) surface, being sawtooth shaped (see Fig. 1). This is only true within the interval of the physical staircase. This infinite mathematical model is matched to reality by modeling a handclap not by a truly spherical wave, but by a wave that only contains propagation directions from the emitter directly to the staircase within the angular interval $[\alpha_1, \alpha_2]$ that assures impingement on the staircase and within the interval $[\alpha_3, \alpha_4]$ if, in addition, reflections on the ground are considered as well. Hence, the handclap is only spherical if observed on the staircase. Whatever sound patterns are emitted to areas outside of the considered intervals is unimportant for the present study. The vectors **d**, **h**, **D**, and **H** are defined in Fig. 1. For $\mathbf{e}_x$ and $\mathbf{e}_z$ being unit vectors along the x, respectively, z direction, straightforward geometrical considerations result in

$$\alpha_1 = \arccos \frac{(\mathbf{h}+\mathbf{d})\cdot \mathbf{e}_z}{\sqrt{h^2+d^2}}, \tag{1}$$

$$\alpha_2 = \arccos \frac{(\mathbf{h}+\mathbf{d})\cdot \mathbf{e}_z}{|\mathbf{h}+\mathbf{d}-\sqrt{D^2+H^2}\mathbf{e}_x|}, \tag{2}$$

$$\alpha_3 = \arccos \frac{(-\mathbf{h}+\mathbf{d})\cdot \mathbf{e}_z}{|-\mathbf{h}+\mathbf{d}-\sqrt{D^2+H^2}\mathbf{e}_x|}, \tag{3}$$

$$\alpha_4 = \arccos \frac{(-\mathbf{h}+\mathbf{d})\cdot \mathbf{e}_z}{\sqrt{h^2+d^2}}, \tag{4}$$

with

$$\begin{pmatrix} d_x \\ d_z \end{pmatrix} = \begin{pmatrix} \cos\xi & \sin\xi \\ -\sin\xi & \cos\xi \end{pmatrix} \begin{pmatrix} |d| \\ 0 \end{pmatrix}, \tag{5}$$

$$\begin{pmatrix} h_x \\ h_z \end{pmatrix} = \begin{pmatrix} \cos\xi & \sin\xi \\ -\sin\xi & \cos\xi \end{pmatrix} \begin{pmatrix} 0 \\ |h| \end{pmatrix}, \tag{6}$$

and

$$\xi = \arctan \frac{H}{D} + \frac{\pi}{2}. \tag{7}$$

The diffraction theory of Claeys *et al.* that is applied here can be found in the literature.[9,11] Nevertheless, some characteristics of that theory are outlined below. The theory is based on the decomposition of the diffracted acoustic field into pure plane waves, which is essentially only allowed whenever the Lipmann conditions[9,11] are fulfilled, stating that the incident wave length must be of the same order of magnitude as the corrugation period and that the corrugation height must not exceed the incident wave length. If these conditions do not hold, then errors will occur in the description of the sound field within the corrugation. Elsewhere the errors will be small, except when the Lipmann conditions are seriously violated of course. Basically, each of the reflected and transmitted wave fields are decomposed into a series of plane waves, each plane wave of order m having a wave vector

$$\mathbf{K}^m = k_x^m \mathbf{e}_x + k_z^m \mathbf{e}_z, \tag{8}$$

with

$$k_x^m = k_x^{inc} + m\frac{2\pi}{\sqrt{2}q}, \tag{9}$$

and k_z^m determined by k_x^m, the material properties of the considered medium and the dispersion relation $k^2=\omega^2/v^2$, omega being the angular frequency and v being the plane wave velocity. The sign of k_z^m is chosen such, as to fulfill the necessity of plane waves to propagate away from the interface and, whenever k_z^m is purely imaginary, the amplitude must decay away from the interface. The continuity conditions demand continuity of normal stress and normal particle displacements on each spot of the pyramid's staircase. It can be found in Claeys *et al.*[9,11] that this leads to a set of equations that is periodical in x, whence the discrete Fourier transform can be applied, resulting in an equal number of equations and unknown amplitudes of all diffracted orders. It can also be found in Claeys *et al.*[9,11] that this discrete infinite set of equations and unknowns can be chopped to a square linear matrix equation that can be solved by a computer.

III. NUMERICAL RESULTS AND DISCUSSION

The following parameters are chosen such as to match the physical reality of the reported experiments[13] at 10 m in front of the pyramid (see Fig. 1). The observer's height is

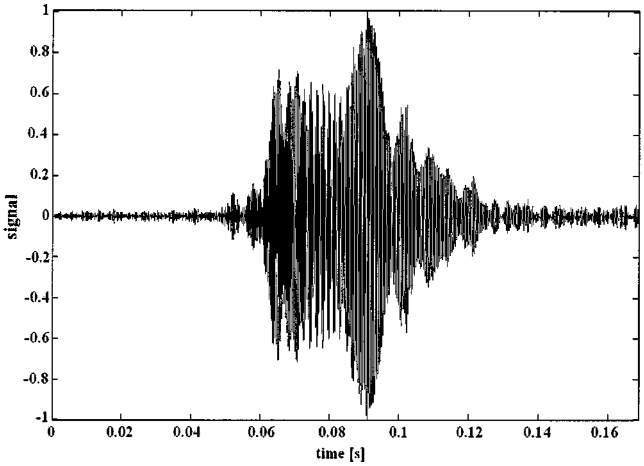

FIG. 2. Normalized calculated direct echo coming from a delta pulse.

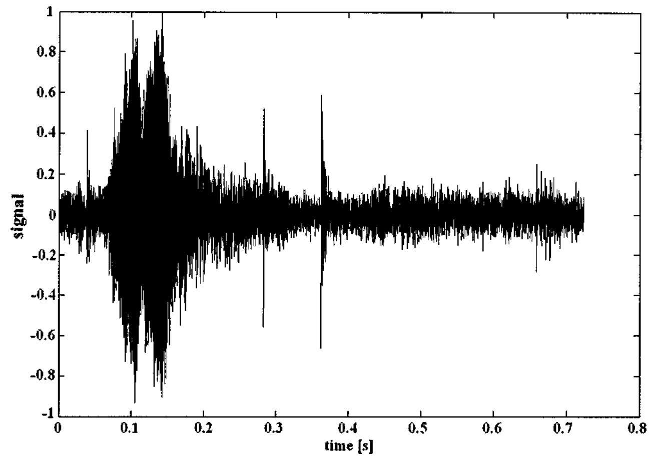

FIG. 3. Normalized recorded signal produced by a Quetzal bird in the forest.

chosen $h=1.80$ m, the observers distance $d=10$ m, the pyramid's dimensions $D=23.84$ m, $H=24.02$ m, $q=0.263$ m. It then follows from (1)–(4) that $\alpha_1=35.01°$, $\alpha_2=78.15°$, $\alpha_3=82.22°$, and $\alpha_4=55.42°$. The material properties in the humid Yucatan air have been taken as $\rho=1.1466$ kg/m^3 for the density and $v=343$ m/s for the sound velocity. Those for the limestone[14] staircase have been taken as $\rho=2000$ kg/m^3 for the density, $v_l=4100$ m/s for the longitudinal wave velocity and $v_s=2300$ m/s for the shear wave velocity. Damping has not been taken under consideration. For the parameters just given, the Lipmann conditions are given as follows: For frequencies lower than 1844 Hz, the numerical simulations will be perfect. For frequencies higher than 1844 Hz, there will be small errors in the description of the sound field within the corrugation, but not elsewhere. For very high frequencies, say more than 5000 Hz, errors may also occur in the prediction of the sound field outside of the corrugation, i.e., in the air and where the observer is situated. The errors gradually grow for higher frequencies and are due to "shadow zones" and neglecting internal reflection within the stairs.

A. Direct echo coming from a delta pulse

Within the angular interval $[\alpha_1,\alpha_2]$, the incident sound is considered to be spherical and contains 500 frequencies equally distributed between 500 and 3000 Hz. All incident plane waves have the same amplitude regardless of their direction and frequency. The former is necessary to produce the spherical wave, the latter is needed to produce a delta function like handclap. The spherical wave is modeled by 300 plane waves propagating along equally distributed angles within the interval $[\alpha_1,\alpha_2]$. There is no serious violation of the Lipmann conditions. Only for frequencies above 1844 Hz can there be some errors in the sound field description within the corrugation, but that is not of significant importance here because we are only interested in effects at the observer's position.

In Fig. 2, the calculated echo as a function of time is given, corresponding to an incident spherical pulse. This signal looks very clean, i.e., there is not too much noise outside of the echo, and is somewhat similar to the normalized plot in Fig. 3 of the actual sound of a real Quetzal bird in the forest. The latter signal was downloaded in*.wav format from the website of David Lubman.[13] The few delta function like peaks in the middle of that latter plot are the result of cracks that can be heard in the recorded sound file and are probably due to wood creaks in the bird's biotope. Figure 4 shows a normalized plot of the pyramid's echo and is obtained from a*.wav file that was also downloaded from Lubman's website.[13] This signal is far from clean. This is primarily due to low frequency noise coming from the interaction of wind with the microphone. Since it is almost impossible to compare sound signals in time–space, it is necessary to study sonograms or spectrograms of the obtained signals.[15] A sonogram depicts the amplitude as a function of time "t" and as a function of frequency "f." It is obtained by a time limited Fourier transform. Here, we used a gaussian window of 0.002 s width. The sonograms are plotted by means of a gamma correction of 2. If the recorded sound is truly and solely an echo that comes from diffraction on the staircase, some patterns that will be mathematically described now, may appear in the sonogram. From (9) an $-m$'th order echo may appear if the following relation holds:

$$k_x^{\mathrm{inc}}=m\frac{\pi}{\sqrt{2}q}. \tag{10}$$

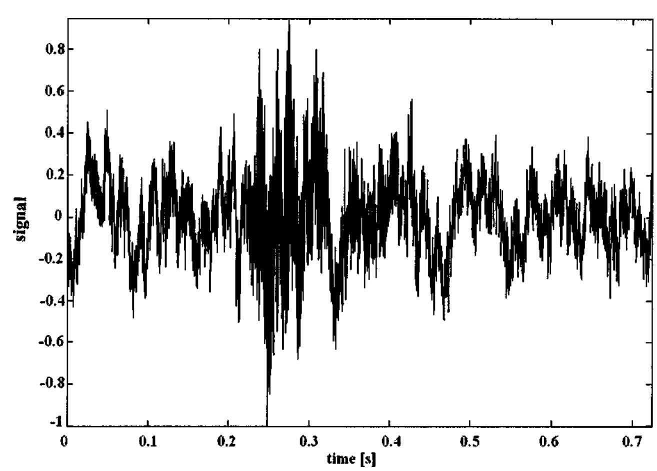

FIG. 4. Normalized recorded signal of the echo coming from the pyramid.

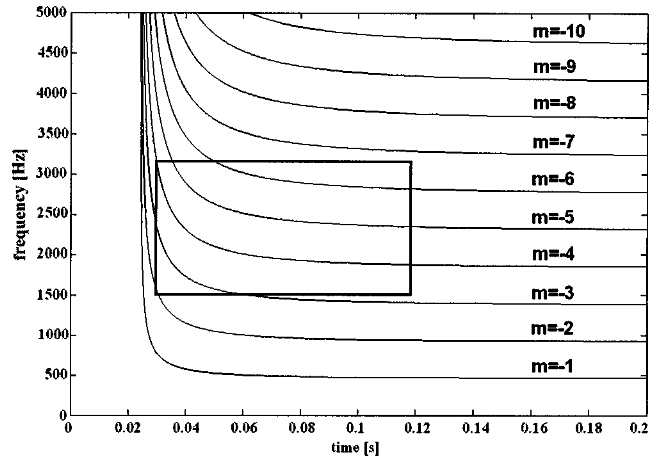

FIG. 5. Bragg diffraction lines on a sonogram. The sonogram shows information as a function of time (horizontal axis) and frequency (vertical axis). The square window is a reference window that represents the same time–frequency values in each sonogram in this report.

If this is combined with the dispersion relation, the angle, as a function of the frequency at which the echo may appear, can be calculated. If a ray-consideration is then applied, the time delay as a function of each angle, taking into account the wave speed in air, can also be obtained. This ultimately results in:

$$t(-m,f) = \frac{|d_z+h_z|}{v\cos\mathrm{Re}\left[\frac{\pi}{2}-\arctan\frac{\sqrt{\left(\frac{2\pi f}{v}\right)^2-\left(m\frac{\pi}{\sqrt{2q}}\right)^2}}{m\frac{\pi}{\sqrt{2q}}}\right]}. \tag{11}$$

In Fig. 5 the curves that are represented by (11) are depicted by means of a sonogram. In all sonograms that are presented here, the vertical axis represents the frequency in the range from 0 Hz (bottom) to 5000 Hz (top). The horizontal axis always spans a range of 0.2 s. However, the instant values on the horizontal axis do not always range from 0 to 0.2 s. It is only the difference between the right side of the horizontal axis and the left side that is 0.2 s. This is of course due to the fact that sound recordings contain no information about the absolute values of the start of recording and the end of recording. However, in order to compare the different sonograms that are presented here, we have taken into account physical considerations like the presence of the handclap in the recordings of Lubman[13] or the knowledge of the time-origin in our calculations, to draw a time–frequency window on each of the presented sonograms that is absolutely the same in each sonogram. This window will therefore function as the reference window for the discussions below. The absolute position of the window is chosen as to contain the relevant information that is present in Fig. 6, which is the sonogram that corresponds to the calculated echo of Fig. 2. This sonogram shows almost the same structure as the one of

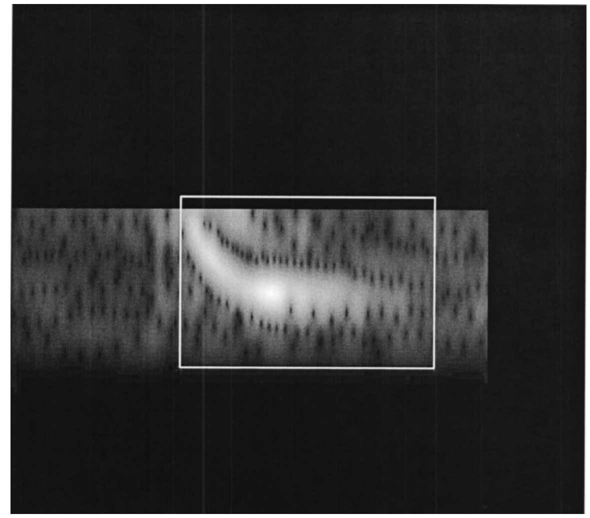

FIG. 6. Sonogram of the calculated direct echo coming from a delta pulse. The axes are equal to those of Fig. 5.

Fig. 7, which corresponds to the recorded Quetzal bird chirp in the woods (see also Fig. 3). The only important difference is the frequency at which the patterns appear and their duration. The actual bird chirps at lower frequencies than the calculated pyramid's echo. The authors do not know how a young Quetzal bird sounds like, but perhaps the resemblance would then be better. If Fig. 5 is compared with Fig. 6, it is noticed that even though the classical grating equation predicts the possibility of elevated amplitude lines in the sonogram, not all lines are associated with a relevant amplitude if the continuity conditions are also taken into account (see Fig. 6). However, the elevated amplitude patterns that do appear correspond more or less to the lines of Fig. 5. Especially there is a strong appearance of the $m=-4$ or $m=-5$ back reflected sound. The fact that it is not simple to decide which order is actually determining the elevated amplitudes is probably due to the interference of several plane waves because the incident sound is spherical. This is slightly in contrast with the assumption of Lubman[5] that the Bragg-orders can

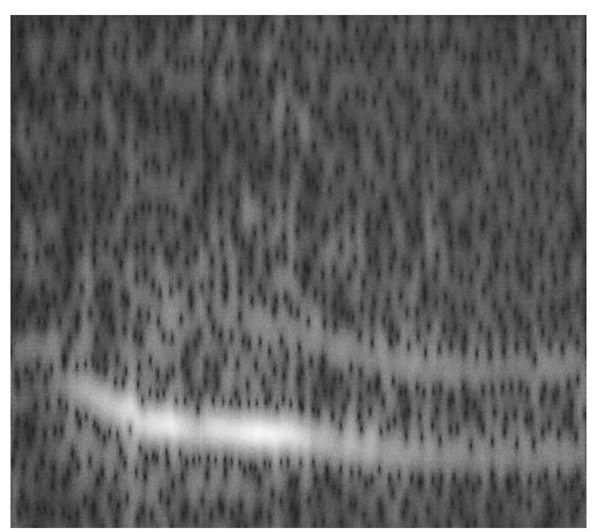

FIG. 7. Sonogram of the recorded Quetzal bird chirp in the forest. The axes are equal to those of Fig. 5.

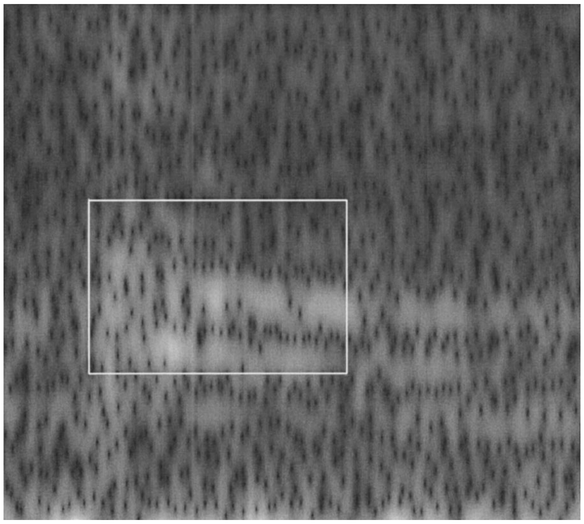

FIG. 8. Sonogram of the recorded echo coming from the pyramid. The vertical axis is equal as in Fig. 5, the horizontal axis spans the same time interval length. The reference window is situated at the same time/frequency values as in Fig. 5.

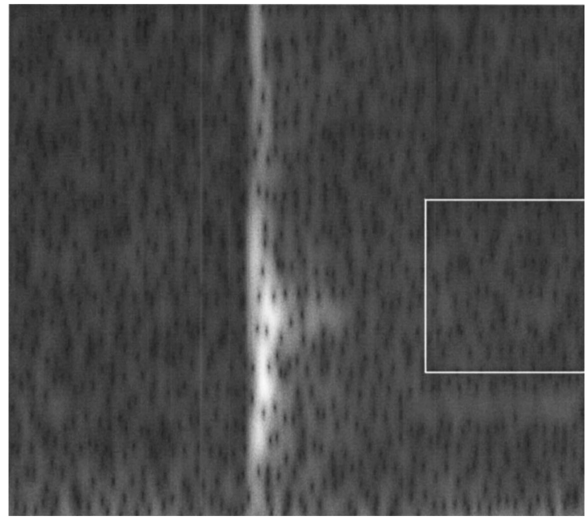

FIG. 9. Sonogram of the recorded (and mathematically isolated) handclap. Same comments on the axes as in Fig. 8.

be well seen in the sonogram of the recorded echo. In order to examine this contradiction, we have calculated the sonogram that actually corresponds to the recorded pyramid's echo of Fig. 4. The result is shown in Fig. 8. Within the reference time/frequency window, the same pattern can be found more or less (if you look through the noise) as in Fig. 6. However, Fig. 8 shows that it is absolutely not for certain that all patterns that are noticeable would correspond to the lines of Fig. 5. There is even something more obscure, which is the presence of "patterns" outside the reference window. If these were simply coming from Bragg diffraction, they would also appear in Fig. 6, where not only the mathematical grating equation is taken into account, but also the continuity conditions. Since they do not appear in Fig. 6 (or have an amplitude which is too small to be noticed), it can already be concluded that these patterns cannot simply be the result of pure Bragg diffraction and that an extra effect must be involved.

B. Direct echo coming from a handclap

The answer to the critical question as to what then actually causes these patterns can be revealed if one considers Fig. 9. The latter figure depicts the sonogram of the handclap taken from the recordings of Lubman[13] and being isolated from the echo of the same recording. A handclap is actually far from a delta function, because not all frequencies have the same amplitude. Actually, the handclap contains several frequency bands. For this purpose we have also simulated the echo resulting from a real handclap instead of a pulse. The handclap itself (as taken from Lubman[13]), which takes 0.02 s and must be followed by 0.18 s of silence in order to get a realistic time window of 0.2 s, needs to be represented by 4096 frequencies in between 5 and 25 000 Hz. Because of the amount of RAM memory needed and due to a limited CPU speed, taking into account all these frequencies in our diffraction procedure would result in a calculation time that exceeds the lifetime of our high speed computer.

This, together with the fact that the higher the frequencies, the more seriously Lipmann's conditions are violated, and a trade off between handclap reproducibility and calculation time, led to the decision to reduce the number of frequencies to 1968 in between 400 and 10 240 Hz. Taking into account higher frequencies would have violated Lipmann's conditions and would have taken us too much time. Consideration of only frequencies up to 5000 Hz led to an incident handclap that didn't sound right and led to an echo that did not at all correspond with reality. The reason of the latter effect is that a complicated handclap is much harder to deal with than the pulse of last section. Whereas a frequency chop for a pulse results in a new pulse that is quickly followed by a period of silence within the 2 s time window of interest, a frequency chop for a handclap results in unnegligible noise following the handclap, which is too strong if only frequencies up to 5000 Hz are considered. This noise, which is less important if frequencies up to 10 240 Hz are taken into account, is also diffracted and due to time shifts may even overlap with neighboring time windows after diffraction. Therefore the numerical echo (as can be seen in Fig. 10), corresponding with an incident numerical handclap with fre-

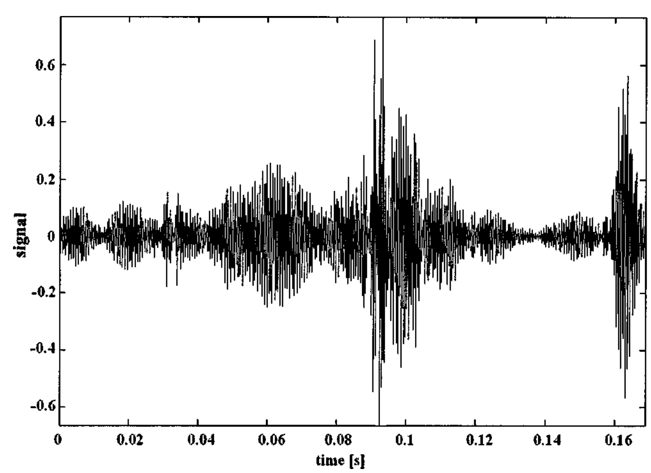

FIG. 10. Calculated direct echo coming from a handclap.

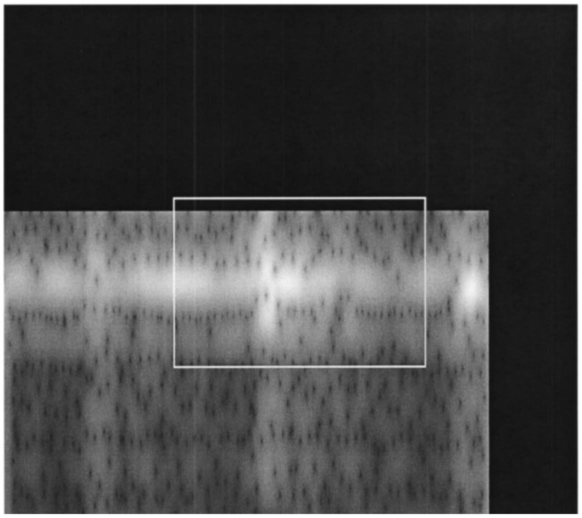

FIG. 11. Spectrogram of the calculated direct echo coming from a handclap.

quencies higher than 10 240 Hz neglected, is, contrary to physical experiments, not limited in time. In Fig. 10, for reasons of calculation time limitations, we have, just as in the previous calculations, considered the results for all applied plane waves at all applied frequencies, but we have only taken into account 1024 positions of time within the interval of interest for reproducing the result. This means that a time limited Fourier transform cannot extract frequencies higher than the sampling frequency of 3034 Hz. However, if we take a look at the sonogram in Fig. 11, which corresponds with the numerical signal in Fig. 10 and is made just like all previous sonograms, we can see 4 frequency bands instead of only 2 in Fig. 6. Even more important is that they coincide with the experimentally measured frequency bands of Fig. 8. Therefore, even if, because of computer limitations, a true temporal description cannot be obtained, still what the frequencies are concerned the simulation reproduces the experimental result obtained by Lubman.[13] This proves that the lower two frequency bands in the experiments are mainly caused by the nature of the handclap and not as much by the diffraction process itself. In other words the echo is a function of the kind of incident sound.

C. Direct and indirect echo coming from a handclap

In Sec. III A we discussed the echo coming from a pulse and showed that the presence of 4 frequency bands in the reflected sound instead of 2 was probably due to the kind of incident sound. In Sec. III B this statement was proved by simulating the echo coming from the handclap in the experiments.[13] Yet another important question that needs to be resolved is the influence of the ground in front of the stairs of the pyramid. Up until now we have neglected this effect. We now consider the extreme condition where the ground is a perfect reflector. Hence sound coming from the handclap is not only propagating strait to the pyramid, but is also reflected on the ground before propagating towards the pyramid. Furthermore sound reflected from the pyramid may be received after strait propagation from the stairs or may again be reflected by the ground before being received. Therefore, the received signal G consists of 4 parts:

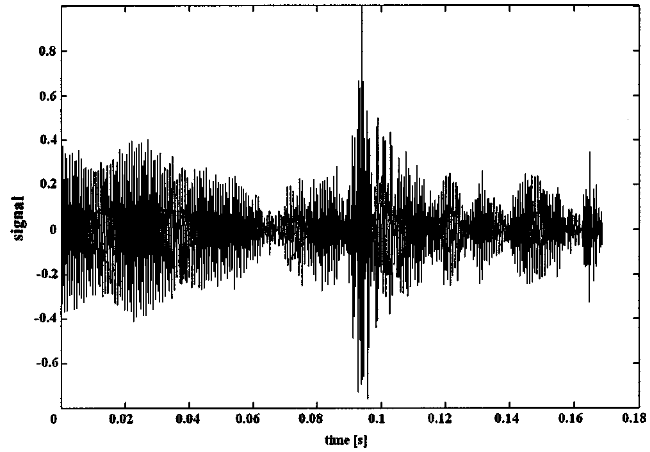

FIG. 12. Calculated direct and indirect echo coming from a handclap.

(i) G_1: Sound traveled directly to the pyramid and being received directly;

(ii) G_2: Sound traveled directly to the pyramid and being received after being reflected by the ground;

(iii) G_3: Sound being reflected by the ground before having traveled to the pyramid and being received directly;

(iv) G_4: Sound being reflected by the ground before having traveled to the pyramid and being received after being reflected by the ground.

We call the person in front of the pyramid "person" and his mirror image (see Fig. 1) the "mirror person." The ground is replaced by a mathematical mirror plane. Mathematically G_1 is emitted by the person and again received by the person. G_2 is emitted by the person and received by the mirror person. G_3 is emitted by the mirror person and received by the person. G_4 is emitted and received by the mirror person. By filling in the correct coordinates of the person $(\mathbf{d}+\mathbf{h})$ and the mirror person $(\mathbf{d}-\mathbf{h})$, simulation is again possible of each signal. Then

$$G=G_1+G_2+G_3+G_4. \tag{12}$$

The result of G can be seen in Figs. 12 and 13. Again these figures cannot really tell anything about the temporal distribution of the frequencies, nevertheless it is seen that the ground has no influence on the presence or absence of the 4 frequency bands. In the future it would be great if someone would do some experiments at the pyramid by placing a reflector or an absorber in front of the staircase in order to see what effect it has on the received echo.

IV. EXPLANATION OF THE RAINDROP EFFECT

If people are climbing the pyramid, their shoes produce sound pulses containing all frequencies. Even though such pulses are more complicated, we model them here by means of a superposition of normally incident pure plane waves. Figure 14 shows the amplitude of the reflection coefficient of the zero order and the −1st order as a function of the frequency. Since we are only interested in understanding the raindrop effect, we focus in Fig. 15 on the frequency zone

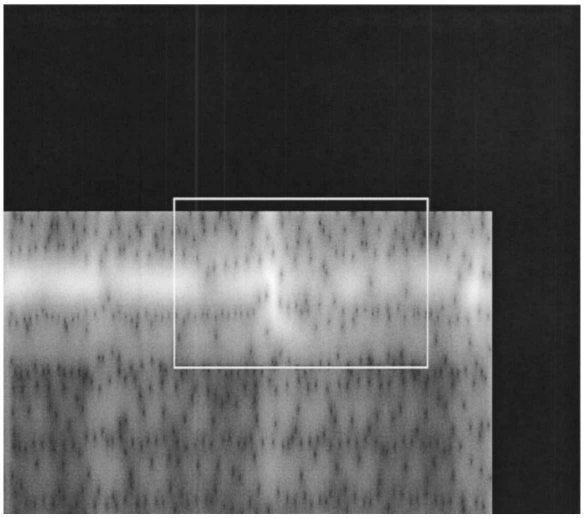

FIG. 13. Spectrogram of the calculated direct and indirect echo coming from a handclap.

where the −1st order reflected sound undergoes a transition from evanescent to bulk waves. That happens at a frequency f given by

$$f=\frac{v}{\sqrt{2}q}=919.57\ \ \text{Hz}. \tag{13}$$

In addition it can be verified with what has been explained above that this transition zone fulfills the Lipmann conditions whence the validity of the numerical calculations cannot be cast doubt on.

On the right side of the transition frequency in Fig. 15, the −1st order reflected sound is as important, regarding its amplitude, as the zero order reflected sound. Furthermore, in Fig. 16, the propagation direction (measured from the pyramid's surface) of the −1st order reflected sound is depicted as a function of the frequency. On the right of and close to the transition frequency, the −1st order diffracted sound travels almost parallel to the pyramid's surface. Now, since that sound is bulk in nature (not evanescent) and since it has

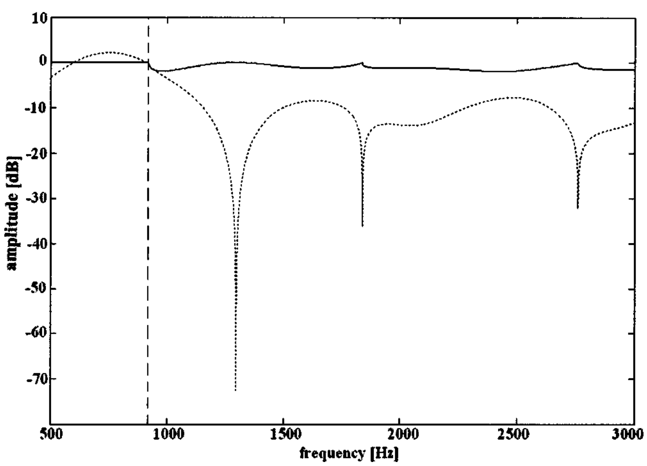

FIG. 14. The zero order reflection coefficient (solid line) and the −1st order reflection coefficient (dotted line) as a function of the frequency, for normal incident sound on the pyramid. The left side of the dashed line corresponds to evanescent −1st order reflected waves, while the right side corresponds to bulk −1st order reflected waves.

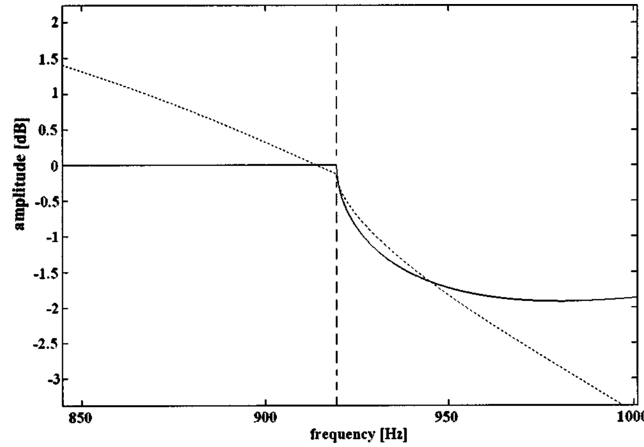

FIG. 15. Close up of Fig. 14.

a considerable amplitude (see Fig. 15), it is actually hearable for the observer seated on the lowest stair step. The observed frequency range is limited since (see Fig. 16) only a limited bunch of frequencies produce sound that can reach the observer's ear, which is situated at small angles from the pyramid's surface. Frequencies between 920 and 1000 Hz indeed sound like the main frequency that is present in the bunch of frequencies generated by a raindrop falling in a bucket filled with water.

V. CONCLUDING REMARKS

It is shown that the echo that is produced by the pyramid consists of diffracted sound coming from the staircase. The echo is formed by a process which is connected with Bragg reflection, but more effects are as important as well, such as the continuity conditions on the stairs and the frequency pattern of the incident sound. Therefore we would be pleased if someone could do some extended experiments in front of the pyramid in order to measure the echo as a function of the incident sound. We would not be surprised if the use of drums or timber wood to produce sound pulses would result in a better echo. The model also showed that the ground in front of the pyramid has no influence on the reflected frequency bands. Nevertheless it could not be shown what the

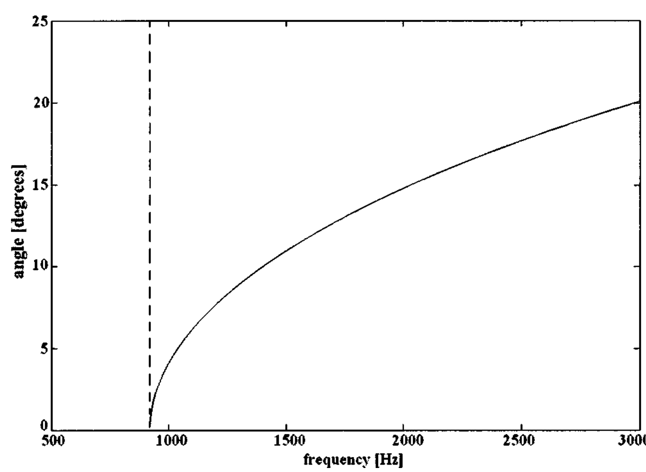

FIG. 16. The propagation angle of the −1st order reflected sound as a function of the frequency, measured from the pyramid's surface.

temporal effect is. It could elongate the echo or shorten it depending on the reflective properties of the ground. It would also be interesting to test the effect of the sound speed in air on the produced echo. This speed can vary in the dry season and wet season and can also vary with temperature. It is also explained how an observer seated on the lowest stair step may hear ‘‘raindrops’’ falling in a water filled bucket when other people are climbing the upper stairs.

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