Study of the scattering of leaky Rayleigh waves at the extremity of a fluid-loaded thick plate

Nico F. Declercq,[a] A. Teklu,[b] and M. A. Breazeale
National Center for Physical Acoustics, University of Mississippi, Oxford, Mississippi 38677

Rudy Briers
Katholieke Hogeschool Zuid-West-Vlaanderen Regentaat en Normaal School Department, Sint Jozefstraat 1, B-8820 Torhout, Belgium

Oswald Leroy
Interdisciplinary Research Center, KU. Leuven Campus Kortrijk, E. Sabbelaan 53, B-8500 Kortrijk, Belgium

Joris Degrieck
Soete Laboratory, Department of Mechanical Construction and Production, Ghent University, Sint Pietersnieuwstraat 41, B-9000 Ghent, Belgium

Gennady N. Shkerdin
Institute of Radio Engineering and Electronics, Russian Academy of Sciences, Vvedensky Sq., 1, Fryzino, 141120, Moscow region, Russia

(Received 17 February 2004; accepted 17 August 2004)

A study by means of the Schlieren technique for the visualization of the ultrasonic beams has revealed that when the leaky Rayleigh waves propagating along the horizontal edge of a thick fluid-loaded solid plate are scattered at the extremity of the plate, they travel around the corner and start leaking into the liquid along the Rayleigh angle measured from the normal to the vertical edge of the plate. Furthermore, the study reveals that the leaky Rayleigh waves are stimulated by the border of an incident ultrasonic-bounded beam more than by the interior of the beam. A comparison with an earlier work shows that the characteristics of the scattering of the leaky Rayleigh waves at the edge of the plate is very different from that of the Scholte–Stoneley waves. © *2004 American Institute of Physics*. [DOI: 10.1063/1.1804618]

I. INTRODUCTION

Rayleigh waves are acoustic eigenmodes of a solid-vacuum interface. They are surface waves involving elliptic polarization of material particles near the interface. Rayleigh waves have a counterpart on solid-liquid interfaces, quite similar, except that they leak energy into the liquid and are therefore called the leaky Rayleigh waves. The leaky Rayleigh waves leak energy at an angle (measured from the normal to the interface) that is called the Rayleigh angle θ^{Rayl}. Accordingly, they can be generated by means of the sound incident at the Rayleigh angle through the mode conversion of the bulk waves into the surface waves. In the literature, a number of papers can be found that deal with the interaction of the Rayleigh waves with a corner, i.e., the extremity of a thick plate.[1–10] Other papers deal with the interaction of the Rayleigh waves with the inhomogeneities and discontinuities.[11–14]

However, all the cited papers study the nonleaky Rayleigh waves, i.e., the Rayleigh waves on a solid-air or solid-vacuum interface. The question arises how the "leaky" Rayleigh waves interact with the extremity of a thick plate. Will they be transmitted and travel around the corner, or will they only be reflected and be accompanied by a radiation field in all directions? Earlier, some papers have been published[15–17] concerning the surface waves on a solid-liquid interface and their interaction with the extremity of a thick plate. Nevertheless, these papers only consider the incident Scholte–Stoneley waves and the nonincident (leaky) Rayleigh waves. The Scholte–Stoneley waves are eigenmodes on a solid-liquid interface and their velocity is smaller than any of the sound velocities in both surrounding media. Therefore, according to Snell's law, they cannot leak energy and are different from the leaky Rayleigh waves. However, it is interesting to discuss the results of Refs. 15–17 for comparison with the results for the leaky Rayleigh waves, as given further in the succeeding paragraphs. For this reason, we first focus on Fig. 1, where the different angles are defined. The angles are measured from the horizontal to the surface in the half space beyond the edge of the plate (see right side of Fig. 1) with the clockwise sense involving the negative angles and the anticlockwise sense involving the positive angles. The angle of incidence is θ^{inc} and the angle of reflection is θ^{r}, whereas the angle of sound coming from the emitter and left undisturbed by the solid is θ^{t}. Whenever an additional sound field is visible, it is given by an angle θ^{s}.

The studies reported in Refs. 15–17 show that the Scholte–Stoneley waves ($\theta^{\mathrm{inc}}=180°$) are primarily scattered

[a]Author to whom correspondence should be addressed; permanent address: Soete Laboratory, Department of Mechanical Construction and Production, Ghent University, Sint Pietersnieuwstraat 41, B-9000 Ghent, Belgium; electronic mail: nicof.declercq@ugent.be
[b]Present address: Department of Physics and Astronomy, College of Charleston, 66 George Street, Charleston, SC 29424.

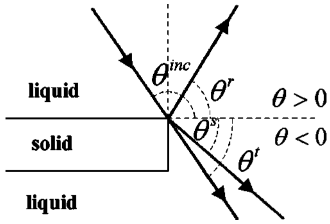

FIG. 1. Definition of the different angles θ^{inc}, θ^{r}, θ^{s}, and θ^{t}. The angles are measured from the horizontal axis left from the plate. The positive angles are anticlockwise, whereas the negative angles are clockwise.

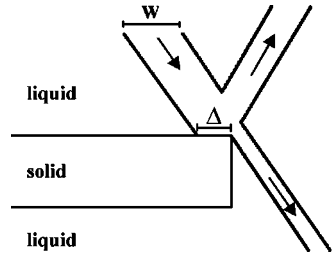

FIG. 2. Definition of Δ: the distance between the first beam edge and the plate edge and W: the horizontal physical width of the beam.

in the forward direction ($\theta^{r}=\theta^{s}=\theta^{t}=0°$). However, the sound is also scattered in all the other directions, but this scattered sound is accompanied by a much smaller amplitude than that in the forward direction. A relatively strong-mode conversion also occurs into the reflected leaky Rayleigh waves. These surface waves radiate (leak) energy into the direction $\theta=90+\theta^{\text{Rayl}}$ with θ^{Rayl} as the Rayleigh angle. In addition, the scattered sound shows a minimum in the $-\theta^{\text{Rayl}}$ direction, which means that the leaky Rayleigh waves are hardly stimulated along the vertical edge of the plate ($\theta=-90°$). Still, the Scholte–Stoneley waves exist both in the transmission (along the vertical edge, i.e., $\theta=-90°$) and in the reflection (along the horizontal edge, i.e., $\theta=180°$). This knowledge, together with the lack of literature concerning the event of the scattering of the leaky Rayleigh waves at the edge of a thick plate formed the reason for studying this phenomenon.

II. EXPERIMENTAL PROCEDURE

We have used the thick solid plates (3-cm high, 5-cm wide, 10-cm long) in order to avoid the Lamb wave stimulation. The Lamb waves are the plate waves that are easily stimulated in the thin plates. At the frequencies we have used in the experiments, 3–6 MHz, a thickness of 3 cm in this regard can be considered as "infinity." Furthermore, the thick plates were polished because we wanted to avoid the secondary effects, such as the additional scattering or diffraction phenomena, caused by the surface roughness.

The incident and diffracted sound are experimentally visualized by means of the monochromatic Schlieren technique. This technique is described in Ref. 18. The ultrasound is generated by apodized quartz transducers, of the same type as in Ref. 18, that produce the Gaussian sound beams.

A leaky Rayleigh wave is generated on the solid-water interface by means of a bounded beam incident at the Rayleigh angle θ^{Rayl}. This angle is determined through the observation of the Schoch effect,[19–21] i.e., the appearance of two reflected amplitude lobes with a null strip in between, for an incidence spot relatively far from the edge of the plate. Then, the plate is moved without altering the incident-bounded beam until the beam reaches the edge of the plate. This configuration is shown schematically in Fig. 2, where W is the physical horizontal width of the beam and where Δ is the distance from the left border of the beam to the extremity of the solid plate. Whenever $\Delta < W$, a fraction of the incident sound beam will propagate without severe disturbance coming from the solid edge; this is the direct and undisturbed transmitted sound. Whenever $\Delta \geqslant W$, there is no direct and undisturbed transmitted sound.

III. RESULTS AND DISCUSSION

A. Results for an aluminum sample

First, we consider an incident 2.5-cm-wide bounded beam of 3 MHz, incident on a brass plate. The dimensions of the plate are given previously. According to Ref. 22, the Rayleigh wave velocity for aluminum is 2906 m/s. This corresponds to a Rayleigh angle θ^{Rayl} of 31° for the velocity of the sound in water being 1480 m/s. It was observed that if $\Delta \approx W$ for $\theta^{\text{inc}}=90°+\theta^{\text{Rayl}}$, in addition to a reflected beam in the direction $\theta^{r}=90°-\theta^{\text{Rayl}}$, a "scattered" sound beam was generated along the direction $\theta^{s}=-\theta^{\text{Rayl}}$. This scattered sound beam was not there for other angles of incidence. The phenomenon for an aluminum plate can be seen in Fig. 3, where $\theta^{\text{Rayl}}=31°$. As soon as $\Delta < W$, there is also a direct, undisturbed transmitted beam visible, as can be seen in Fig. 4 ($\Delta \approx 0.78\, W$). Now, because $\theta^{s}=-\theta^{\text{Rayl}}$ for the scattered sound beam and because this beam disappears as soon as the incidence angle differs from $\theta^{\text{inc}}=90°+\theta^{\text{Rayl}}$, it is likely that the beam is actually the energy leakage coming from the leaky Rayleigh wave propagating along the vertical edge of the thick plate, i.e., along $\theta=-90°$ and stimulated by the interaction of the leaky Rayleigh wave on the horizontal side of the plate with the edge of the plate. We believe that this interpretation is more reasonable than the interpretation that the sound beam would be radiated by the incident leaky Rayleigh wave when reaching the extremity of the plate. Hence,

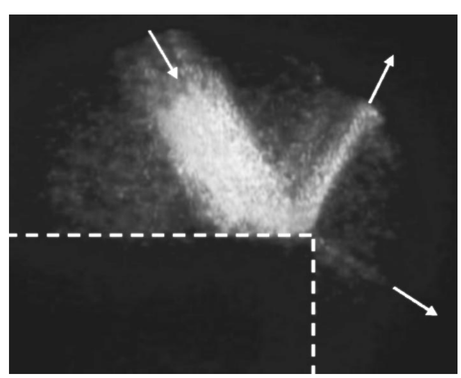

FIG. 3. Incidence of 3 MHz, 2.5- cm-wide bounded beam on aluminum. $W \approx 2.9$ cm, $\Delta \approx W$ (Schlieren picture). The dashed lines show where the plate is situated in the experiment. The arrows show the sound propagation direction.

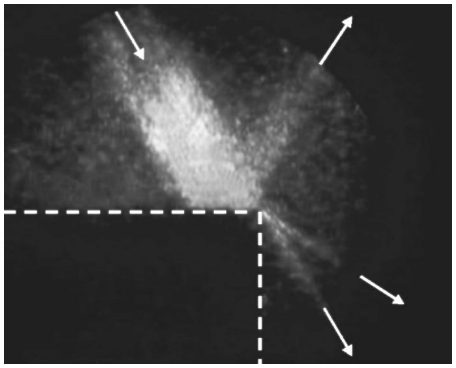

FIG. 4. Incidence of 3 MHz, 2.5-cm-wide bounded beam on aluminum. $W \approx 2.9$ cm, $\Delta \approx 0.78\ W$ (Schlieren picture). The dashed lines show where the plate is situated in the experiment. The arrows show the sound propagation direction.

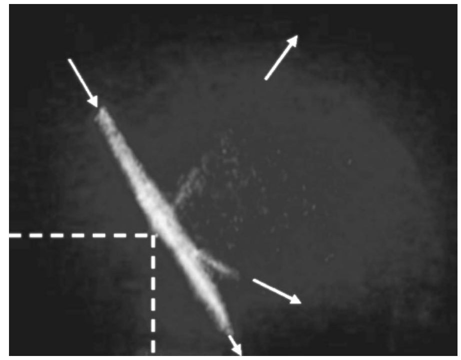

FIG. 5. Incidence of 6 MHz, 0.6-cm-wide bounded beam on aluminum. $W \approx 0.7$ cm, $\Delta \approx 0.2\ W$ (Schlieren picture). The dashed lines show where the plate is situated in the experiment. The arrows show the sound propagation direction.

the incident leaky Rayleigh wave reaches the extremity of the plate and partly propagates around the corner onto the vertical edge of the plate. While doing so, it keeps leaking its energy and that is what is visible as a "bounded beam" in the direction $\theta^s = -\theta^{\mathrm{Rayl}}$.

As a matter of fact, the effect of "surface wave propagation around the corner" is reasonable because the leaky Rayleigh waves are elliptically polarized. This elliptical particle motion at the corner stimulates the elliptical motions on the vertical edge. That is also the reason why the Scholte–Stoneley waves can travel around the corner.[15–17] However, it is noticed that the characteristics of the scattering of the leaky Rayleigh waves is much different from the scattering of the Scholte–Stoneley waves at the corner. Whereas, the Scholte–Stoneley waves scatter mostly in the forward direction ($\theta = 0$), the leaky Rayleigh waves scatter mostly in the direction $\theta = -90°$, resulting in a leakage field in the $\theta^s = -\theta^{\mathrm{Rayl}}$ direction.

Remark that this phenomenon is invisible whenever $\Delta > W$. This is probably due to the leaky feature of the generated surface waves. When they have leaked too much energy into the liquid, their amplitude when propagating around the corner is too small to generate a leaky field in the direction $\theta^s = -\theta^{\mathrm{Rayl}}$ that is visible by the Schlieren imaging technique.

B. The beam edges are responsible for leaky Rayleigh wave generation

Because the generated bounded beam at the angle $\theta^s = -\theta^{\mathrm{Rayl}}$ is directly proportional to the amplitude of the leaky Rayleigh wave propagating along the vertical edge and therefore also directly proportional to the amplitude of the leaky Rayleigh wave propagating along the horizontal edge in the region of incidence, it is possible to reveal what part of the incident beam is mostly responsible for the stimulation of the leaky Rayleigh waves. If it is the center of the beam, then the amplitude of the sound beam at $\theta^s = -\theta^{\mathrm{Rayl}}$ must increase whenever Δ increases from very small to $W/2$. In Fig. 5, Δ is very small ($\Delta \approx 0.2\ W$) and becomes larger in Fig. 6 ($\Delta \approx 0.6\ W$). Nevertheless, no important amplitude change, except for a small amplitude drop, is visible in the bounded beam along the direction $\theta^s = -\theta^{\mathrm{Rayl}}$. This means that the center of the beam does not add any noticeable energy to the Rayleigh wave. In other words, the leaky Rayleigh wave is stimulated by the borders of the bounded beam. In fact, this is not really surprising in the framework of the inhomogeneous wave theory, where it is known[23–26] that the leaky Rayleigh waves are stimulated not by the homogeneous plane waves but by the inhomogeneous plane waves. As a matter of fact, a bounded beam is a physical entity whose behavior can be simulated in the different frameworks of the bounded beam models. The most famous model is the Fourier model, where a bounded beam is formed by means of a superposition of plane waves.[25,26] A less famous model is the superposition of the inhomogeneous waves.[25] Nevertheless, those descriptions are global descriptions, where only a global resulting effect can be simulated, such as the Schoch effect. Those models cannot describe what happens locally to the sound when it is incident on an interface between two different media. Locally, it is clear that the border, contrary to the center, of a bounded beam is more similar to an inhomogeneous plane wave than to a homogeneous plane wave. As can be seen in Fig. 7, the border of a Gaussian beam has a profile quite similar to that of an inhomogeneous wave, i.e., the exponential amplitude growth/decay along the wave front, whereas the center of the beam has properties similar to the homogeneous plane waves. Because the inhomogeneous waves are better suited to stimulate the Rayleigh

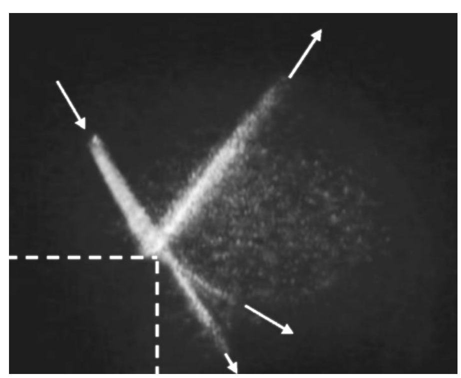

FIG. 6. Incidence of 6 MHz, 0.6-cm-wide bounded beam on aluminum. $W \approx 0.7$ cm, $\Delta \approx 0.6\ W$ (Schlieren picture). The dashed lines show where the plate is situated in the experiment. The arrows show the sound propagation direction.

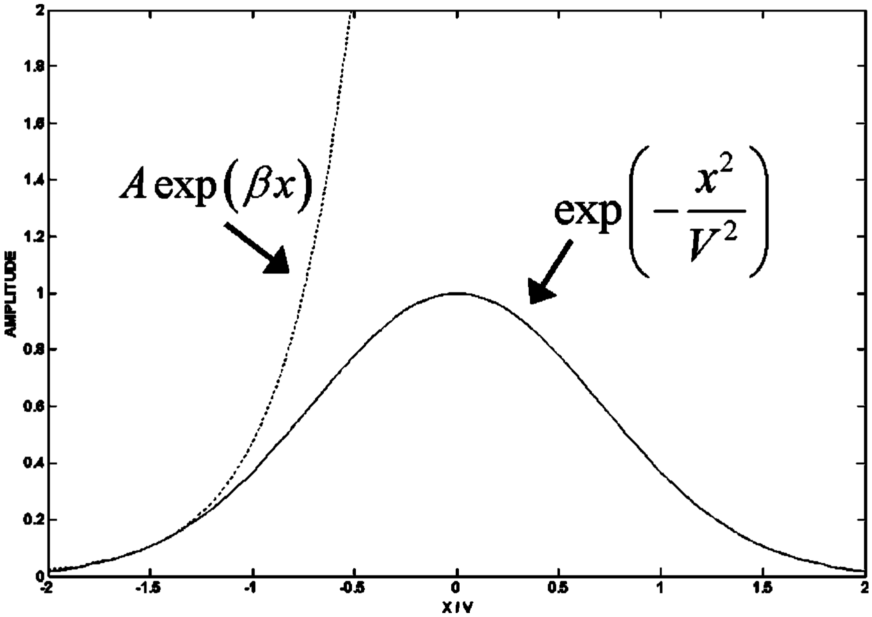

FIG. 7. The edges of a Gaussian beam have a profile that is relatively similar to that of an inhomogeneous wave. This is probably the reason why the edges of a Gaussian beam stimulate the leaky Rayleigh waves. The x axis is directed along the wave front. The value β is called the inhomogeneity of the accompanied inhomogeneous wave.

waves, it is understandable that the edge of the bounded beam is more important than the center for that cause. From the theory of the inhomogeneous waves, it is also known that some inhomogeneities are better capable of stimulating the Rayleigh waves than others. Everything depends on the kind of interface and on the frequency. It is therefore likely that the beam profile and also the beam width are the important parameters. For example, if a bounded beam would be used that is not Gaussian but that has an exponentially varying amplitude profile with bounded edges, then the center is not homogeneouslike but inhomogeneous and the whole beam will be responsible for the stimulation of the Rayleigh waves. This is the effect of the bounded inhomogeneous waves.[25,27]

C. Results for a brass sample

Until now, we have only discussed the results for an aluminum sample. In order to make sure that our conclusions do not just hold for aluminum but also for a different solid, we have repeated the experiments for a brass plate. According to Ref. 22, the Rayleigh wave velocity for brass is 1964 m/s. This corresponds to a Rayleigh angle θ^{Rayl} of 49° for the velocity of sound in water being 1480 m/s. One configuration is shown in Fig. 8, for a 0.6-cm-wide bounded beam of 6 MHz, incident on a spot defined by $\Delta \approx 0.8\, W$, where it can be verified that the physical phenomenon remains unchanged except that here, $\theta^{\mathrm{Rayl}}=49°$. In fact, contrary to the phenomenon on aluminum, here, $|\theta^s| > |\theta^t|$.

D. The disappearance of the Schoch phenomenon

It is also worthy to note that the Schoch displacement, as it appears whenever sound is incident at the Rayleigh angle on a spot relatively far from the edge of the plate, is disturbed when the incidence spot is near the corner and even seems to disappear whenever $\Delta \leq W$. This is of course because the Schoch phenomenon is due to the interaction between the reflected bounded beam and the sound field coming from the generated leaky Rayleigh waves. Whenever the extremity of the plate is reached, the leaky Rayleigh waves are scattered by the edge and are partly propagating along the vertical edge. Now, the interesting part of this phenomenon is that the leaky Rayleigh waves leak energy around the corner that does not interfere anymore with the directly reflected sound beam. Hence, the 'bounded' beam that is generated along the direction $\theta^s = -\theta^{\mathrm{Rayl}}$ is a pure leakage field. Nevertheless, it is not a pure inhomogeneous wave because it results from a Gaussian incident beam and not from an exact distinct inhomogeneous wave.

The disappearance of the Schoch phenomenon when $\Delta > W$ is best seen when Fig. 8 is compared with Fig. 9. In both figures, the angle of incidence is the same and so are the frequency and beam width; however, the Schoch phenomenon is only visible in Fig. 9, where the incidence spot is relatively far from the corner, and not in Fig. 8.

IV. CONCLUSIONS

It is shown that the leaky Rayleigh waves partly propagate around the corner of a thick solid plate. It is also shown

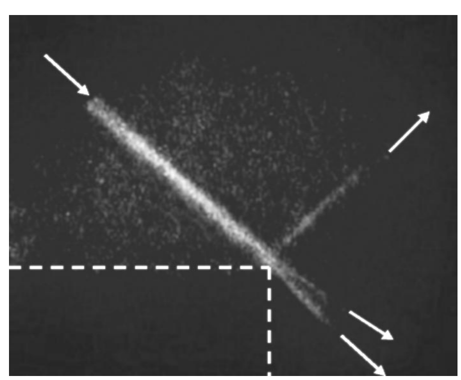

FIG. 8. Incidence of 6 MHz, 0.6-cm-wide bounded beam on brass. $W \approx 0.9$ cm, $\Delta \approx 0.8\, W$ (Schlieren picture). The dashed lines show where the plate is situated in the experiment. The arrows show the sound propagation direction.

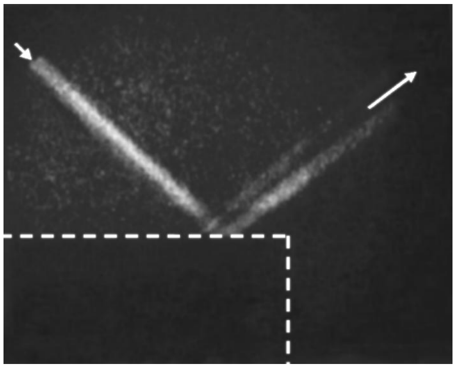

FIG. 9. Incidence of 6 MHz, 0.6-cm-wide bounded beam on brass. $W \approx 0.9$ cm, far from the edge, the Schoch phenomenon is visible. (Schlieren picture). The dashed lines show where the plate is situated in the experiment. The arrows show the sound propagation direction.

that the leaky Rayleigh waves are physically generated by the borders of the incident beam and not so much by the center. As far as we know, there are no other experiments described in the literature that have revealed this feature. This shows that only the beam borders are responsible for the physical generation of the leaky Rayleigh waves, whereas the mathematical models that describe the bounded beams by means of a superposition of the homogeneous or inhomogeneous infinite plane waves ascribe responsibility for the leaky surface wave generation to the complete area occupied by the bounded beam. Consistent experimental results are given for aluminum and brass. A comparison with the cited earlier reports shows that whenever along the vertical edge at the extremity of a plate, the leaky Rayleigh waves need to be generated, it is better to use the incident leaky Rayleigh waves than the Scholte-Stoneley waves. This might be very important for the nondestructive testing of materials.

ACKNOWLEDGMENTS

This work was supported by the Collaborative Linkage Grant of NATO (PST.NR.CLG.980315) and by the Flemish Institute for the Promotion of the Scientific and Technological Research in Industry (I.W.T.).

[1]T. Yoneyama and S. Nishida, J. Acoust. Soc. Am. **59**, 206 (1976).
[2]Z. L. Li, J. D. Achenbach, I. Komsky, and Y. C. Lee, ASME Trans. J. Appl. Mech. **59**, 349 (1992).
[3]V. V. Krylov, Sov. Phys. Acoust. **331**, 86 (1987).
[4]V. V. Krylov and V. G. Mozhaev, Sov. Phys. Acoust. **31**, 457 (1985).
[5]V. G. Mozhaev and V. V. Krylov, Vestn. Mosk. Univ., Ser. 3: Fiz., Astron. **26**, 76 (1985).
[6]D. A. Bruttomesso, L. J. Jacobs, and C. Fiedler, J. Nondestruct. Eval. **16**, 21 (1997).
[7]A. K. Gauetesen, Wave Motion **35**, 91 (2002).
[8]D. A. Bruttomesso and L. J. Jacobs, J. Eng. Mech. **125**, 111 (1999).
[9]J. Kane and J. Spence, Geophysics **28**, 715 (1963).
[10]B. V. Budaev and D. B. Bogy, Wave Motion **22**, 239 (1995).
[11]V. V. Popov, Acoust. Phys. **40**, 577 (1994).
[12]V. V. Popov, Acoust. Phys. **40**, 377 (1994).
[13]B. R. Tittmann, L. A. Ahlberg, and A. K. Mal, Appl. Phys. Lett. **49**, 1333 (1986).
[14]J. Pohl, Materialpruefung **43**, 446 (2001).
[15]A. Tinel, Ph.D. thesis, Université le Havre 1991.
[16]R. Briers, O. Leroy, and G. N. Shkerdin, J. Acoust. Soc. Am. **101**, 1347 (1997).
[17]A. Tinel and J. Duclos, J. Acoust. Soc. Am. **95**, 13 (1994).
[18]M. A. Breazeale, J. Opt. A, Pure Appl. Opt. **3**, S1 (2001).
[19]A. Schoch, Acustica **2**, 1 (1952).
[20]A. Schoch, Acustica **2**, 18 (1952).
[21]W. Neubauer, in *Physical Acoustics*, edited by W. P. Mason and R. N. Thurston (Academic Press, New York, 1973), Vol. 10, p. 61.
[22]G. W. C. Kaye and T. H. Laby, *Tables of Physical and Chemical Constants*, 14th ed. (Longman, London, 1973).
[23]B. Hosten and M. Deschamps, Acustica **59**, 193 (1986).
[24]G. Quentin, A. Derem, and B. Poirée, J. Acoust. **3**, 321 (1990).
[25]N. F. Declercq, R. Briers, J. Degrieck, and O. Leroy, IEEE Trans. Ultrason. Ferroelectr. Freq. Control (to be published); J. Acoust. Soc. Am. **115**, 2624 (2004).
[26]H. L. Bertone and T. Tamir, Appl. Phys. **2**, 157 (1973).
[27]N. F. Declercq, J. Degrieck, and O. Leroy, J. Acoust. Soc. Am. **116**, 51 (2004); **113**, 2283 (2003).