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The representation of 3D gaussian beams by means of inhomogeneous waves

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Abstract

There are different methods to mathematically represent a bounded beam. Perhaps the most famous method is the classical Fourier method that consists of the superposition of pure homogeneous plane waves all traveling in different directions and having an amplitude that can be found by the Fourier transform of the required profile. This method works perfectly for 2D as well as for 3D bounded beams. However, some researchers prefer the inhomogeneous wave theory to represent a bounded beam because some phenomena, e.g. the Schoch effect, are explained by this method by means of concepts that agree better with intuition. There are several papers dealing with this method for 2D gaussian beams. Until now, it has never been considered possible to represent 3D gaussian beams as well. The present paper shows a method to overcome this shortcoming and presents different sorts of 3D gaussian beams that are built up by means of inhomogeneous plane waves.

Keywords: Bounded beams; Gaussian beams; Inhomogeneous waves; Complex waves

1. Introduction

There are several ways of mathematically representing bounded beams. The most common technique is the Fourier method [1] in which a bounded beam is built up by pure plane waves all incident at different angles. One of the other methods is the decomposition of a bounded beam into inhomogeneous waves [2,3]. Inhomogeneous waves differ from pure plane waves in that their wave vector is complex, involving an exponentially decaying/growing amplitude along their wave fronts. If a bounded beam is imagined to be built up by inhomogeneous waves, one approaches the bounded beam by means of a summation of inhomogeneous waves in a limited interval. Beyond the limits of that interval, exponentially growing tails appear [2,3]. The approach has always been understood as a least squares approximation and applies the so-called Prony technique [2] and the orthogonality of Laguerre polynomials. This method, which was first applied on bounded beams by Claeys and Leroy [2], has always been applied to 2D gaussian beams, involving polynomials containing one single variable. The extension to 3D gaussian beams has never been performed since a similar reasoning would involve polynomials containing two variables. As until now, one has never found a promising technique to deal with these polynomials in this context, 3D gaussian beams remained non-existing in inhomogeneous wave theory. The present communication reports a simple technique to represent 3D gaussian-like beams by means of inhomogeneous waves. The technique is based on a fitting procedure in a limited number of spots and not on a least squares approximation on a certain interval.

2. Representing 3D Gaussian-like beams

The profile of a 3D gaussian-like beam is given by

$$f(x,y) = \exp\left(-\frac{x^2}{W^2} - \frac{y^2}{V^2}\right) \tag{1}$$

and is decomposed into inhomogeneous waves as

$$f(x,y) = \sum_{m=-M}^{M} \sum_{n=-N}^{N} A_{m,n} \exp(\boldsymbol{\beta}_{m,n} \cdot \mathbf{r}) \tag{2}$$

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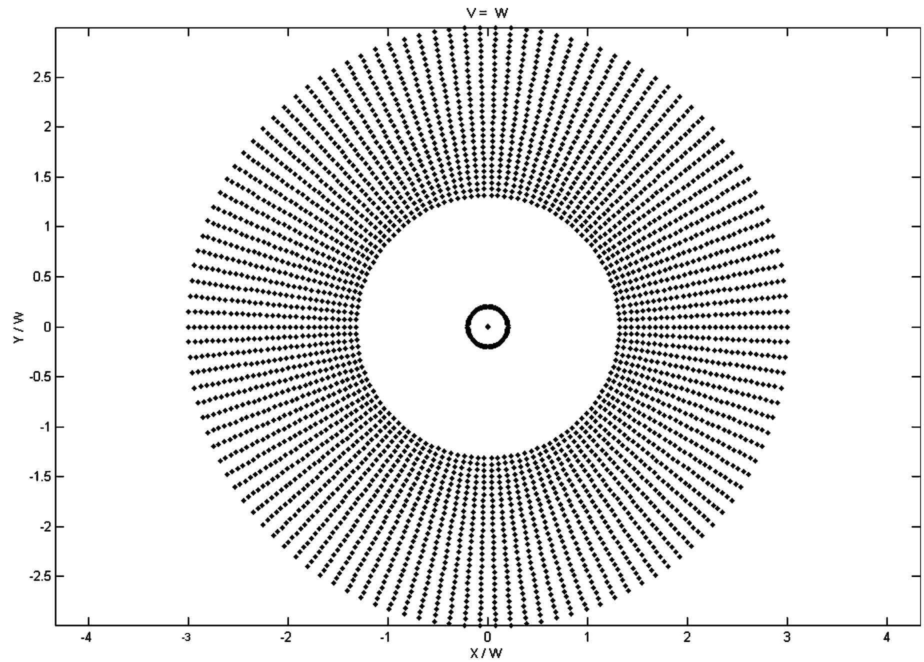

Fig. 1. The distribution of spots where the fit procedure (2) is performed for a perfectly gaussian beam profile ($V = W$).

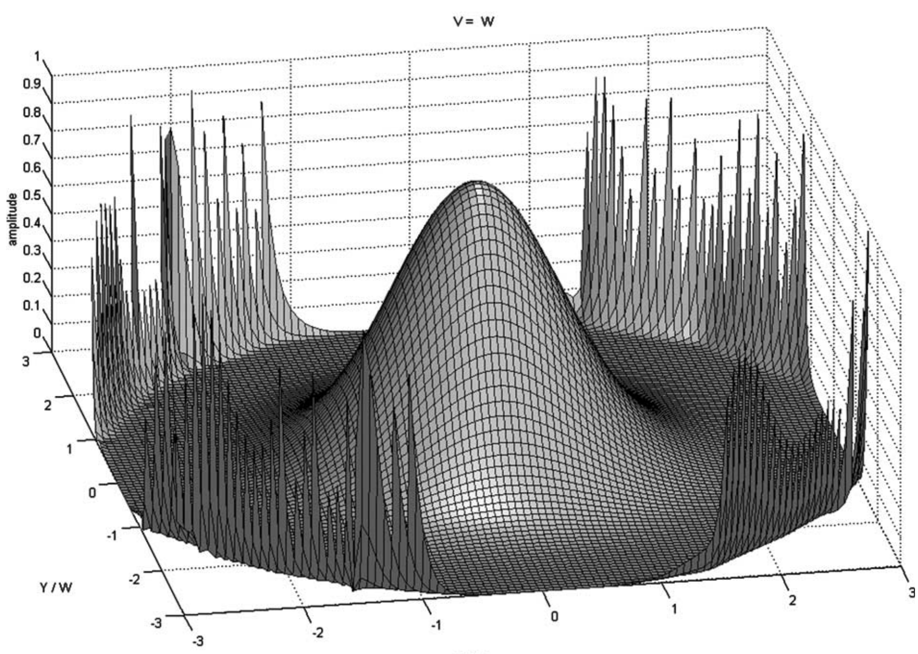

Fig. 2. The profile of a 3D gaussian beam approached by a superposition of inhomogeneous waves ($V = W$).

The amplitude attributed to the inhomogeneous wave of number (m, n) is $A_{m,n}$, the inhomogeneity is $\boldsymbol{\beta}_{m,n}$ with

$$\boldsymbol{\beta}_{m,n} = \beta^x_m \mathbf{e}_x + \beta^y_n \mathbf{e}_y \tag{3}$$

In accordance with the method of Claeys and Leroy [2,3], we apply

$$\beta^x_m = \frac{m}{p_x} \quad \text{and} \quad \beta^y_n = \frac{n}{p_y} \tag{4}$$

with p_x and p_y real numbers.

Furthermore, we choose $(2M + 1) \times (2N + 1)$ spots in the xy-plane in which we demand (2) to hold. Due to the smoothness of exponential functions, Eq. (2) will

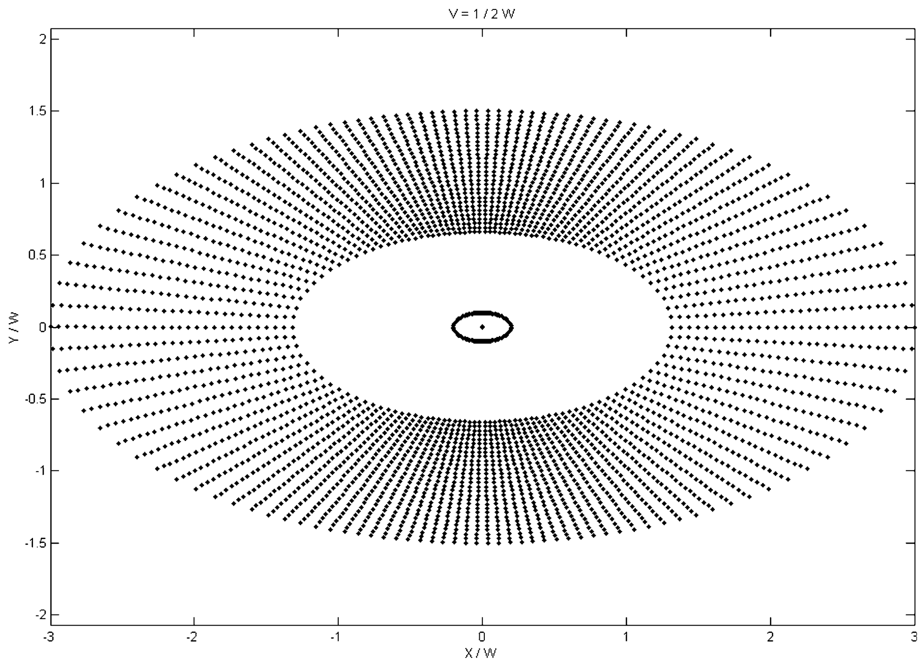

Fig. 3. The distribution of spots where the fit procedure (2) is performed for a gaussian like beam profile ($V = W/2$).

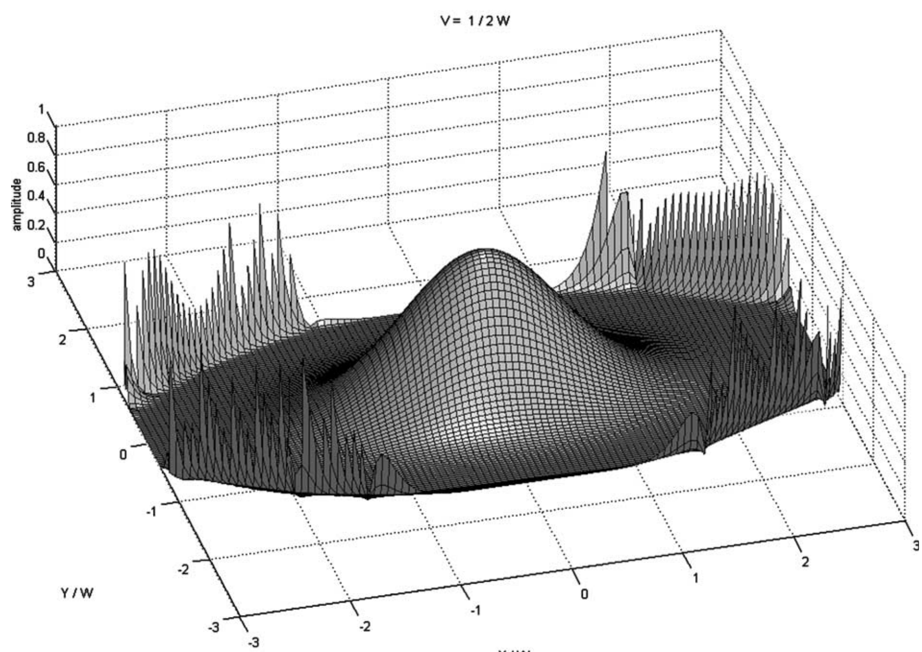

Fig. 4. The profile of a 3D gaussian-like beam approached by a superposition of inhomogeneous waves ($V = W/2$).

hold rather well for intermediate points if the $(2M+1)\times(2N+1)$ spots are chosen well thought out. It is found that almost perfect results are found if the spots are chosen to lay equidistantly on concentric ellipses defined by

$$\frac{x^2}{r^2W^2}+\frac{y^2}{r^2V^2}=1 \tag{5}$$

for r determined by

$$\frac{r}{W}\in\{0.001, 0.2\}\cup\{1.2,\ldots,\mu\} \tag{6}$$

Fig. 1 shows the considered spots for a 3D gaussian beam profile defined by $V = W$. μ is chosen to be 3. In Fig. 2, the obtained gaussian profile is shown by means of a superposition of inhomogeneous waves for

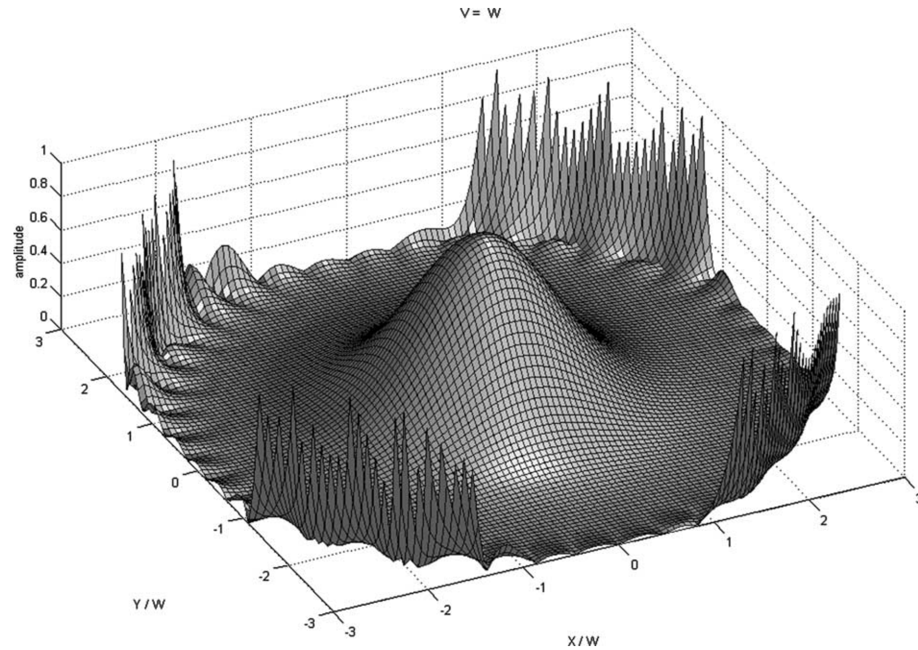

Fig. 5. Same result as in Fig. 2, except that a much smaller number of inhomogeneous waves is used ($M = N = 10$).

$p_x = p_y = 6.7$ and for $M = N = 30$. It is seen that the approximation is promising within a certain radius. Beyond that radius, exponential tails appear in a similar way as in approximations of 2D gaussian beams [2,3]. As is already supposed in (1), the method can also be used if the bounded beam is not perfectly gaussian, but is characterized by $W \neq V$. The chosen spots for the situation $V = \frac{W}{2}$ are depicted in Fig. 3. In Fig. 4, the approximation is shown by a superposition of the same inhomogeneous waves as in Fig. 2, except of course that the coefficients $A_{m,n}$ are different.

Special attention must be paid to the number of inhomogeneous waves that is used, because the larger the number of waves, the larger the number of spots in which the fitting procedure is performed. The latter results in an enhanced overall beam profile approximation. In Fig. 5, the approximation of Fig. 2 is shown for $M = N = 10$. It is clear that wrinkles appear which are not visible in Fig. 2.

3. Conclusion and prospects

A method is presented to approximate 3D gaussian beams by a superposition of inhomogeneous waves. The method seems to work very well, especially if the number of inhomogeneous waves is large. In future, research has to be performed as to investigate how steady this method is for utilization in scattering phenomena, because it is known from the application of 2D gaussian beams that numerical stability is critical [2,3].

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References

[1] N.F. Declercq, R. Briers, O. Leroy, The use of polarized bounded beams to determine the groove direction of a surface corrugation at normal incidence, the generation of surface waves and the insonification at Bragg-angles, Ultrasonics 40 (1–8) (2002) 345–348.

[2] J.M. Claeys, O. Leroy, Reflection and transmission of bounded sound beams on halfspaces and through plates, J. Acoust. Soc. Am. 72 (1982) 585–590.

[3] K. Van Den Abeele, O. Leroy, Complex harmonic wave scattering as the framework for investigation of bounded beam reflection and transmission at plane interfaces and its importance in the study of vibrational modes, J. Acoust. Soc. Am. 93 (1993) 308–323.