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The use of polarized bounded beams to determine the groove direction of a surface corrugation at normal incidence, the generation of surface waves and the insonification at Bragg-angles

Nico F. Declercq *, Rudy Briers, Oswald Leroy

Interdisciplinary Research Center, Katholieke Universiteit Leuven Campus Kortrijk, E. Sabbelaan 53, 8500 Kortrijk, Belgium

Abstract

Zero order reflected sound from a singly corrugated interface between a solid and a liquid, insonified from the solid side by circular polarized shear waves, can become almost perfect linearly polarized in a direction parallel or perpendicular to the corrugations, depending on the frequency, and can therefore reveal the direction of the corrugations.

When narrow bounded beams, formed by a summation of infinite plane waves, are diffracted at certain frequencies, depending on the angle of incidence, or vice versa, one can predict phenomena like backscattering at Bragg-angle incidence and also the creation of Scholte–Stoneley waves.

Keywords: Corrugated interface; Groove direction; Surface waves; Bounded beams; Bragg-angle

1. Introduction

Consider an interface between a solid and a liquid, that is insonified from the solid side, with corrugations parallel to the y-axis, that is described by

$$p(x,z) = f(x) - z = p(x + \Lambda, z) = 0 \tag{1}$$

with Λ the period of the corrugation. The plane in which the diffracted orders are spread can reveal the direction of the grooves, using two transducers, while the polarization of the echo can also reveal the direction using only one transducer, i.e. when circularly polarized normal incident shear waves are used. The latter is discussed here. The results found for polarization will not differ if bounded beams were used instead of plane waves. If however, we wish to describe the complete amplitude distribution in space after diffraction, we must also reckon with bounded beams. The latter is worked out for normal incidence at the frequency that generates a second order Scholte–Stoneley surface wave and for oblique incidence at the Bragg-angle.

* Corresponding author. Address: Department of Mechanical Construction and Production, Faculty of Engineering, Ghent University, Sint Pietersnieuwstraat 41, 9000 Ghent, Belgium.

E-mail address: nicof.declercq@rug.ac.be (N.F. Declercq).

2. Normal incident elliptically polarized plane shear waves

2.1. Theoretical considerations

Plane waves, having a polarization in a plane parallel to the surface generally have a horizontal y-component as well as a vertical x-component, whence a normally incident shear polarized wave field is characterized by its displacement field

$$\mathbf{u}_i = [B\mathbf{e}_x + C\mathbf{e}_y] \exp[\mathrm{i}(z\eta_i - \omega t)] \tag{2}$$

which corresponds to a potential

$$\boldsymbol{\psi}_i = \left[A\mathbf{e}_y - \mathrm{i}\frac{C}{\eta_i}\mathbf{e}_x\right] \exp[\mathrm{i}(z\eta_i - \omega t)] \tag{3}$$

if we rewrite B as $B = -\mathrm{i}A\eta_i$. η_i is the wave number of the incident wave, while ω is its angular frequency.

The x-component of Eq. (2) or the y-component of Eq. (3) corresponds to a vertically polarized normally incident shear wave with continuity conditions developed by Mampaert and Leroy [1,2], while the y-component of Eq. (2) corresponds to a horizontally polarized normally incident shear wave with continuity conditions developed by Declercq et al. [3].

If B and C are in phase, Eq. (2) describes linearly polarized waves, while elliptically if they are out of

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phase. The particular case $C/B = e^{i\pi/2}$ involves circularly polarized anticlock wise rotating incident waves.

The horizontal component of the incident field generates horizontal reflected waves [3], while the vertical component generates vertical reflected waves [1,2], whence the zero order reflected transversal sound is described by the following potential

$$\boldsymbol{\psi}_r = \left[-\mathrm{i}\frac{CP_0}{\eta_i}\mathbf{e}_x + \frac{\mathrm{i}B}{\eta_i}S_0\mathbf{e}_y \right] \exp[\mathrm{i}(z\eta_{s0} - \omega t)] \tag{4}$$

or displacement

$$\begin{aligned}\mathbf{u}_r &= [D\mathbf{e}_x + E\mathbf{e}_y]\exp[\mathrm{i}(z\eta_{s0} - \omega t)] \\ &= [BS_0\mathbf{e}_x + CP_0\mathbf{e}_y]\exp[\mathrm{i}(z\eta_{s0} - \omega t)] \end{aligned} \tag{5}$$

with η_{s0} the wave number of the zero order reflected wave and S_0 and P_0 complex amplitudes that follow from the mode conversion theory of diffraction [1–3].

The values of D and E determine the polarization of the reflected wave, just as A and B do for the incident wave.

2.2. Calculations

All calculations are performed using a sine shaped stainless steel–water interface, described in Table 1.

We consider a normally incident anticlock wise circularly polarized plane wave having unit amplitude

$$\mathbf{u}_i = [\mathbf{e}_x + e^{i\pi/2}\mathbf{e}_y]\exp[\mathrm{i}(z\eta_i - \omega t)] \tag{6}$$

that generates [1–3] a zero order reflected plane wave described in (5). In Fig. 1, the intensity of D and E are plotted as a function of the frequency, while *phase(E)–phase(D)* is plotted in Fig. 2. It is seen in Fig. 1, that both $|D|^2$ and $|E|^2$ show a distinct minimum, i.e. a frequency position at which Scholte–Stoneley waves [1] are generated (minimum in $|D|^2$ at 8.4 MHz), or Love waves [3] (minimum in $|E|^2$ at 8.88 MHz). From Fig. 2, we learn that the displacement rotation of the reflected wave is opposite to that of the incident wave for low frequencies and switches its rotation sense whenever a threshold frequency is surpassed at which surface waves are generated. For each frequency, one can draw an ellipse that is the displacement in real space at each instant of time during one vibration period. The special cases of 8.4 and 8.88 MHz are shown in Fig. 3, where it is seen that at the Scholte–Stoneley wave generating frequency, the reflected wave has a polarization that is almost linear and parallel to the wrinkles on the surface, while at the Love frequency the same effect happens but now the polarization is perpendicular to the wrinkles. Hence this reveals a method for determining the direction of the corrugation.

Table 1
Characteristics of the considered stainless steel–water interface

	Stainless steel	Water
ρ (kg/m^3)	7300	1000
v_{shear} (m/s)	3100	–
v_{long} (m/s)	5700	1480
$\Lambda = 350$ μm		
Max(f) = 30 μm		

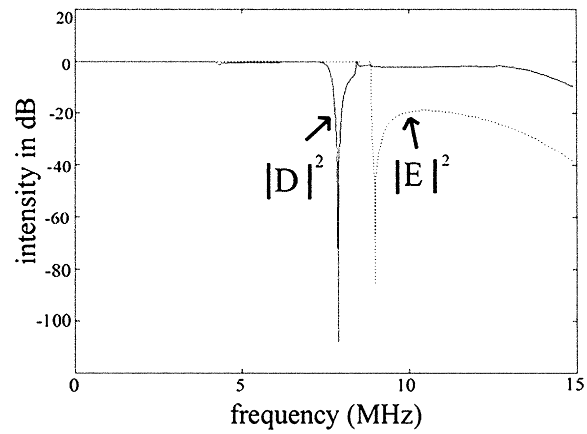

Fig. 1. The intensity spectrum of E and D.

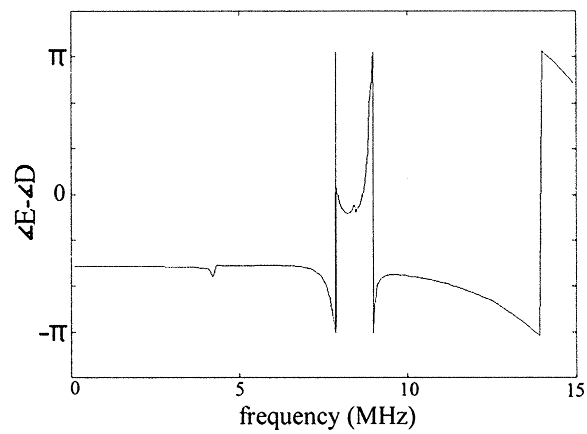

Fig. 2. The spectrum of the phase difference between E and D.

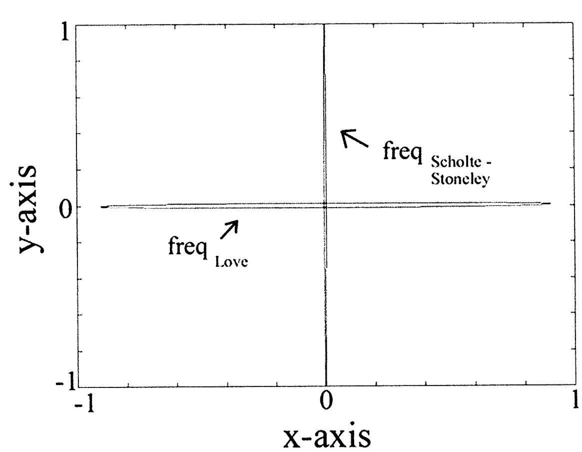

Fig. 3. The almost linear ellipses that are the displacements in one period of time, at the 'Scholte–Stoneley frequency' and the 'Love frequency'. Remember that the grooves in the surface are directed along the y-axis.

3. Incident gaussian beams

3.1. Theoretical considerations

Now let us consider an incident beam that is gaussian bounded in one dimension, i.e. in the XZ-plane.

We denote the incoming gaussian beam as

$$\Psi_i(\mathbf{r}) = \int e^{-i\mathbf{k}\cdot\mathbf{r}}\phi_i(\mathbf{k})d\mathbf{k} = \frac{k}{2\pi N}\int \exp[-ik(x\sin\theta + z\cos\theta)] \times \exp\left[\frac{-k^2(1-\cos(\theta-\theta_0))}{\sigma_k^2}\right]d\theta \quad (7)$$

with a spatial beam width

$$\sigma = \frac{1}{\sigma_k}\sqrt{\frac{\pi}{2}} \quad (8)$$

and angular beam width

$$\sigma_\theta = \arcsin\sqrt{-\frac{\pi\ln(1/2)}{\sigma^2 k^2}} \quad (9)$$

The angular beam width is defined as the angle $\theta - \theta_0$ for which

$$\exp\left[\frac{-k^2(1-\cos(\theta-\theta_0))}{\sigma_k^2}\right] = \frac{1}{2} \quad (10)$$

while the spatial beam width is defined as the x-value for which $\exp(-x^2/(2\sigma^2)) = 1/2$.

3.2. Calculations

3.2.1. Bragg-angle incidence

We consider longitudinally polarized sound (velocity v_l) incoming from the solid side and we solely reckon with the longitudinally polarized reflected sound. In agreement with diffraction of electromagnetic plane waves [4–6], we define the Bragg-angle for a certain frequency as the angle of incidence for which most of the incoming energy is first order back scattered, or

$$\mathbf{k}_i = -\mathbf{k}_1 \iff \sin\theta_i = -\sin\theta_{1l} \quad (11)$$

whence the classical grating equation gives

$$\theta_{\text{Bragg}} = \arcsin\left(\frac{-v_l}{2\Lambda \times \text{freq}}\right) \quad (12)$$

The Bragg-angle can only exist if

$$\text{freq} \geqslant \frac{v_l}{2\Lambda} \quad (13)$$

Relation (12) is a very practical tool for determining Λ from θ_{Bragg}, i.e. the angle at which the source receives the strongest echo.

In Fig. 4, one can not only see that the reflected bounded beam is very divergent (due to the very narrow incident beam), but also that at the Bragg frequency, most of the reflected energy is sent back in the direction of the incident field.

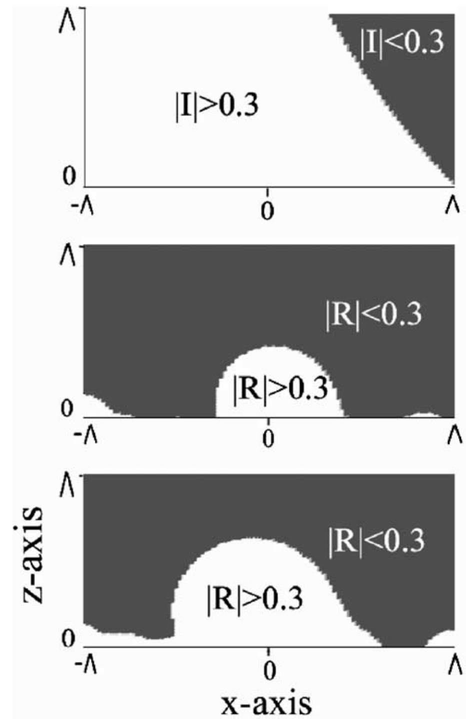

Fig. 4. Amplitude $|I|$ of the incident longitudinally polarized gaussian profiled sound (top) and $|R|$ of the longitudinally polarized reflected sound field, for an angle of incidence of $-35.03°$, for an arbitrary frequency of 10.34 MHz (middle) and for the Bragg frequency, calculated using (12), of 14.18 MHz (bottom). In the white region, the amplitude exceeds an arbitrary chosen value of 0.3, while in the dark region the amplitude is <0.3. Despite of the divergence of the narrow reflected beam, it is clear that there is backscattering at the Bragg frequency (bottom).

There is no immediate relation between the here defined Bragg-angle and the existence of surface waves on a corrugated interface.

3.2.2. Normal incidence at the second order Scholte–Stoneley wave generating frequency

We know [1,2] that at certain frequencies, normal incident longitudinal plane waves can generate Scholte–Stoneley waves. In Fig. 5, one can see that this also occurs when a bounded beam is used. Near the surface, amplitudes are calculated that exceed the incoming unit amplitude and this amplitude decreases exponentially for larger values of $|z|$. It is interesting to notice the maxima and minima along the x-axis, as a result of forward and backwards traveling surface waves.

4. Conclusions

We have shown a method for discovering the wrinkle direction by using normal incident circularly polarized

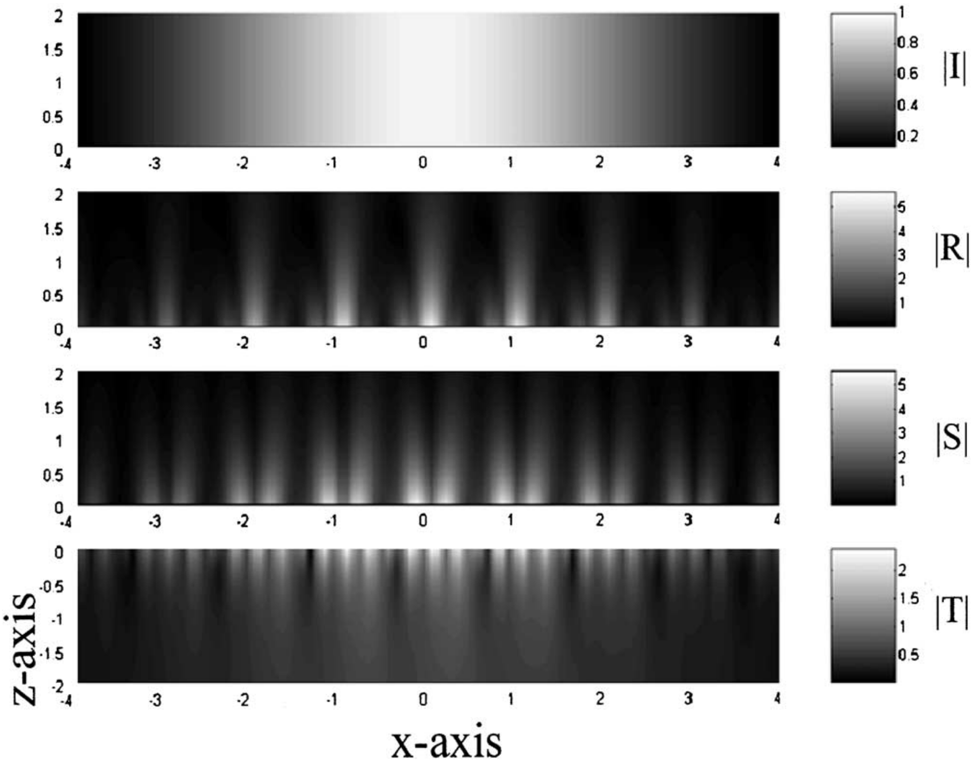

Fig. 5. $|I|$ is the amplitude of the normal incident longitudinal polarized gaussian profiled sound, $|R|$ of the reflected longitudinal, $|S|$ of the reflected shear and $|T|$ of the longitudinal transmitted sound. The frequency is 8.3 MHZ and the generation of second order Scholte–Stoneley waves occurs. This phenomenon does not occur at arbitrary frequencies. Notice the interference of the forward and backwards traveling surface wave. (x-coordinate: in numbers of Λ, z-coordinate: in numbers of $2\max[f(x)]$).

sound and we have shown the existence of Bragg-angles and critical phenomena, i.e. surface wave generation, when gaussian bounded beams are used.

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